QUESTIONS FROM PROBLEM SESSION

1. T. Koberda

Let Γ be a finite simplcial graph with at least two vertices, which is both connected and anti-connected, so that $A(\Gamma)$ splits as neither a direct nor a free product. The extension graph Γ^e can be viewed as

$$\Gamma^e = \bigcup_{g \in A(\Gamma)} \Gamma^g,$$

where here Γ^g denotes the conjugate $g^{-1}\Gamma g$. Under the assumptions, Γ^e is connected and has infinite diameter.

If $G < A(\Gamma)$ is a subgroup then we can form the *restricted extension graph*

$$\Gamma^e_G = \bigcup_{g \in G} \Gamma^g.$$

Question 1. *Is it true that G is either a nontrivial free product or is generated by elliptic elements of $A(\Gamma)$?*

Here, an elliptic element of $A(\Gamma)$ is one which is not a loxodromic isometry of Γ^e, or equivalently its centralizer is not cyclic. It is not difficult to show that Γ^e_G is connected then G is generated by elliptic elements.

The motivation comes from the study of one–ended subgroups of mapping class groups. If $G \to A(\Gamma) \to \mathrm{Mod}(S)$ is a sequence of injections, then the image of G in $\mathrm{Mod}(S)$ can only hope to be convex cocompact if G is free. If the answer to the question is "yes" then if G does not decompose as a free product, its image in $\mathrm{Mod}(S)$ not only contains reducible mapping classes but is in fact generated by reducible mapping classes.

2. E. Bering

Let G be a finitely presented group, let $\{g_1, \dots, g_n\} \subset G$, and let

$$\langle g_1, \dots, g_n \rangle = H < G.$$

The membership problem asks whether given an element $g \in G$, can we decide if $g \in H$? For right-angled Artin groups, the membership problem for finitely generated subgroup of $A(\Gamma)$ is solvable when Γ is a tree, and it is unsolvable when Γ contains an induced square.

Question 2. *Is it true that the membership problem for finitely generated subgroups of right-angled Artin groups solvable if and only if Γ has no induced squares?*

By a result of Kambites, Γ has an induced square if and only if $A(\Gamma)$ contains a product of two nonabelian free groups.

Bridson produced a right-angled Artin group containing a finitely presented subgroup with unsolvable membership problem, but the defining graph has an induced square.

3. A. Kent

Let M be acylindrical with totally geodesic boundary. The space $AH(M)$ of conjugacy classes of discrete faithful representations of $\pi_1(M)$ to $\mathrm{PSL}_2(\mathbb{C})$ is bounded, and its interior is homeomorphic to Teichmüller space and is hence a cell.

Question 3. *If $AH(M)$ locally connected?*

There is an extension of the skinning map to $AH(M)$ to the Teichmüller space of the boundary of M which is finite-to-one.

Question 4. *If $I \subset \partial AH(M)$ is an arc, what can the image of I under the skinning map be?*

4. C. Abbott

Let $AH(G)$ denote the set of acylindrically hyperbolic actions of a group G, which is known to form a partially ordered set.

Question 5. *Is the poset of acylindrically hyperbolic actions a lattice (in the sense of existence of greatest lower bounds and least upper bounds)?*

As a special case, let G act on hyperbolic metric spaces S and R acylindrically, so that elements $g, h \in G$ are loxodromic with respect to the action on S and R respectively. Is there a third hyperbolic metric space T and an acylindrical action of G on T such that both g and h act loxodromically?

5. D. Futer

Remarks on the simple loop question: let $\Gamma = \pi_1(S)$, where S is a closed orientable hyperbolic surface, and let $\mathcal{G}$ denote some class of groups. Suppose $\phi\colon \Gamma \to G \in \mathcal{G}$ is a homomorphism with nontrivial kernel. Does there exist a simple loop in the kernel?

The simple loop question is known to have a positive solution when $\mathcal{G}$ is the class of surface groups, by a famous result of Gabai. The case when $\mathcal{G}$ denotes the class of hyperbolic 3–manifold groups of finite volume is a well–known open problem. For $\mathrm{PSL}_2(\mathbb{C})$, the answer is known to be no, and even for homomorphisms into $\mathrm{PSL}_2(\mathbb{R})$.

Question 6. *What is the answer to the simple loop question for the class of right-angled Artin groups? What about mapping class groups of surfaces?*

6. S. Lawton

Let G be a complex reductive Lie group, K a choice of maximal compact subgroup of G, and let Γ be a finitely presented group. We have an inclusion of character varieties $X_\Gamma(K) \subset X_\Gamma(G)$. We say that Γ is flawed if there exists a strong deformation retraction from $X_\Gamma(G)$ to $X_\Gamma(K)$.

Conjecture 7. *Right-angled Artin groups are flawed.*

It is known that free products of cyclic groups, abelian groups, nilpotent groups, and generalized torus groups are all flawed. Closed surface groups in genus two or more are not flawed.

Some partial results for right-angled Artin groups are known, such as a deformation retract to $\mathrm{Hom}(\Gamma, G(K))//G$.