Probability Seminar

Organizer: Christian Gromoll & Tai Melcher

Friday, 1:00–2:00pm, Kerchof 317

Feb 1 **Vasileios Maroulas**, UNC

Small noise large deviations for infinite dimensional stochastic dynamical systems

Freidlin-Wentzell theory, one of the classical areas in large deviations, deals with path probability asymptotics for small noise stochastic dynamical systems. For finite dimensional stochastic differential equations (SDE) there has been an extensive study of this problem. In this work we are interested in infinite dimensional models, i.e. the setting where the driving Brownian motion is infinite dimensional. In recent years there has been lot of work on the study of large deviations principle (LDP) for small noise infinite dimensional SDEs, much of which is based on the ideas of Azencott (1980). A key in this approach is obtaining suitable exponential tightness and continuity estimates for certain approximations of the stochastic processes. This becomes particularly hard in infinite dimensional setting where such estimates are needed with metrics on exotic function spaces (e.g. Hölder spaces, spaces of diffeomorphisms, etc.).

Our approach to the large deviation analysis is quite different and is based on certain variational representation for infinite dimensional Brownian motions. It bypasses all discretizations and finite dimensional approximations and thus no exponential probability estimates are needed. Proofs of LDP are reduced to demonstrating basic qualitative properties (existence, uniqueness and tightness) of certain perturbations of the original process. The approach has now been adopted by several authors in recent works to study various infinite dimensional models such as stochastic Navier-Stokes equations, stochastic flows of diffeomorphisms, SPDEs with random boundary conditions.

As a first example of this approach, we consider a class of stochastic reaction-diffusion equations, which have been studied by various authors. We establish a large deviation principle under conditions that are substantially weaker than those available in the literature. We next study a family of stochastic flows of diffeomorphisms that arise in certain image analysis problems. Large deviations for the case where the driving noise is finite dimensional has been studied by Ben Arous and Castell (1995). We extend these results to an infinite dimensional setting and apply them to a problem of image analysis.