finite temperature phase diagram of a polarized fermion condensate

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1 Outline

- The two component fermi gas
  \( \sim \) BCS and BEC limits

- The effect of ‘magnetization’
  \( \sim \) What happens when \( N_\uparrow \neq N_\downarrow \)?

- The finite temperature phase diagram
  \( \sim \) and a surprise
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  \[ \sim \text{BCS and BEC limits} \]

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  \[ \sim \text{What happens when } N^\uparrow \neq N^\downarrow? \]

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  ... and a surprise
A brief history of condensates
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- 1925 Einstein points out BEC phenomenon
  - ... [$^4$He, superconductivity, neutron stars, $^3$He, diquark condensates...]

- 1995 BEC realized in trapped atomic gases

- 2004 Fermion condensates in $^6$Li, $^{40}$K
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\[ T_c = \alpha \frac{\hbar^2}{mk} n^{2/3} \quad \text{with} \quad \alpha \equiv \frac{2\pi}{[\zeta(3/2)]^{2/3}} \]
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For \( T < T_{\text{BEC}} \)

\[
\frac{N_0}{N} = \left( 1 - \frac{T}{T_{\text{BEC}}} \right)^{3/2}
\]
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4 Fermion condensates?

Same condition \( \lambda_{dB}^3 n \sim 1 \) gives degenerate fermi system

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\lambda_{dB} \sim \lambda_F, \text{ or } T \sim T_F \equiv \frac{p_F^2}{2m}, \quad p_F = \frac{\hbar}{\lambda_F}
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Two fermions $\sim$ boson
$T_{\text{BEC}}$

BEC of molecules (superfluid)

2\textsuperscript{nd} order

Bound state forms

"Cooper pairs"

Strong attraction

Weak attraction

$T$
5 The BCS-BEC crossover
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(a) BEC superfluidity of bound molecules
(b) BCS - BEC crossover
(c) BCS superfluidity of Cooper pairs
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Diagram showing the transition between BEC and BCS regimes.
6 Theoretical description

\[ H = \int d^3 x \left[ \sum_s \psi^\dagger_s \left(-\nabla^2/2m - \mu\right) \psi_s + g\psi^\dagger_\uparrow \psi^\dagger_\downarrow \psi_\downarrow \psi_\uparrow \right] \]
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\[ |\text{BCS}\rangle = \prod_\mathbf{k} \left( u_\mathbf{k} + v_\mathbf{k}a^\dagger_{\uparrow \mathbf{k}}a^\dagger_{\downarrow -\mathbf{k}} \right) |0\rangle \]
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\[
|\text{BCS}\rangle_{2N} = \mathcal{A} \prod_{i=1}^N \varphi \left( a_i - b_i \right) \quad \varphi(k) = \frac{v_k}{u_k}
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Pairing for any \( g < 0 \) (attraction)

\[ \frac{m}{4\pi a_s} = \frac{1}{g} + \int \frac{d^3p}{(2\pi)^3} \frac{m}{p^2} \]
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\( 1/k_F a \) is dimensionless parameter. \( \rightarrow -\infty \) (BCS); \( \rightarrow \infty \) (BEC)
7 Cooper pairing \((T = 0)\)
Cooper pairing ($T = 0$)

Minimize variational energy
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\[ \Delta = g \langle \psi_\downarrow \psi_\uparrow \rangle_\Delta \]

\[ = g \int \frac{d^3 p}{(2\pi)^3} \frac{\Delta}{2\sqrt{\xi_p + |\Delta|^2}}, \quad \xi_p = \frac{p^2}{2m} - \mu \]
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▷ BCS limit \((1/k_F a \to -\infty)\)

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\Delta_{\text{BCS}} \propto E_F e^{-\pi/2|k_F a|}
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▷ BEC limit \((1/k_F a \to \infty)\)

\[
\mu = -\frac{E_b}{2} = 1/2ma^2 \gg \Delta \sim \text{Schrödinger for pairs.}
\]

\(\sim \varphi(r)\) is bound state wavefn.
What happens if $N_{\uparrow} > N_{\downarrow}$?
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[no traps, just the S/N phase boundary]
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$$M = N_{\uparrow} - N_{\downarrow}$$

‘magnetization’
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Introduce conjugate $h$ ‘magnetic field’

$$H_{\mu,h} = H - \mu N - hM$$
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Bear in mind we are interested in fixed \( M \), not \( h \)!
9 Quasiparticles
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Tree-level effective potential

\[
\frac{S^{(0)}[\Delta_0]}{\beta V} = \frac{1}{g} |\Delta_0|^2 - T \int \frac{d^3p}{(2\pi)^3} \sum_{s=\pm} \ln \cosh \left[ \frac{E_p - \text{sh}}{2T} \right]
\]

\[
E_p = \sqrt{\xi_p^2 + |\Delta_0|^2}.
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Free energy of \((p, s)\) fermionic quasiparticles with

$$E_{p,s} = E_p - sh$$

Interpretation simple in BEC limit
9 Quasiparticles

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Interpretation simple in BEC limit
10 Sarma state and 1st order transition

At $T = 0$ no qp's if $h < E_g \equiv \min_p E_p$
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BCS limit: $E_g = \Delta$
10 Sarma state and 1st order transition

At $T = 0$ no qp’s if $h < E_g \equiv \min_p E_p$

BCS limit: $E_g = \Delta$

Everything scales with $\Delta$
\[ F(\Delta) = \alpha |\Delta|^2 + \beta |\Delta|^4 + \gamma |\Delta|^6 \cdots \]
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Landau theory

First order possible for \( \beta < 0 \)

BCS limit: \( \left( \frac{T_t}{\Delta}, \frac{h_t}{\Delta} \right) = (0.3188, 0.6061) \)
We are interested in fixed $N_{\uparrow}, N_{\downarrow}$!

First order transition $\sim$ phase separation
We are interested in fixed $N_{\uparrow}$, $N_{\downarrow}$!

First order transition $\sim$ phase separation

\[ m(h) \]

\[ m_N, m_S \]

\[ h_c, h \]

\[ T \]

\[ S, N \]

\[ PS \]
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First order transition $\sim$ phase separation

$\Delta$ At $T \neq 0$ thermal qps $\sim$ nonzero magnetization
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First order transition $\sim$ phase separation

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We are interested in fixed $N_{\uparrow}, N_{\downarrow}$!

First order transition $\sim$ phase separation

- At $T \neq 0$ thermal qps $\sim$ nonzero magnetization
- What happens away from BCS limit?
11 Magnetization in the crossover ($T = 0$)

$h > E_g$ without destroying $S$ \(\leadsto\) quasiparticles enter superfluid
Magnetization in the crossover ($T = 0$)

$h > E_g$ without destroying $S$ \quad \sim \quad \text{quasiparticles enter superfluid}

Sheehy and Radzihovsky, 2005
11 Magnetization in the crossover ($T = 0$)

$h > E_g$ without destroying S $\sim$ quasiparticles enter superfluid

Expansion of tree level potential $\sim$ tricritical point at

$$1/k_F a = 2.36799 \quad h/E_F = 6.87592$$
12 Finite temperature
12 Finite temperature

PUNCHLINE –
12 Finite temperature

PUNCHLINE –

They lie on a single tricritical line in \((1/k_F a, m, T)\)!
13 Phase diagram – mean field
$^3\text{He} - ^4\text{He}: \text{a paradigmatic BF mixture}$
15 Finite temperature: technology
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Free energy at one loop

\[
\frac{S^{(1)}[\Delta]}{\beta V} = - \int \frac{d^3 q}{(2\pi)^3} T \sum_{\omega_m} \ln \Gamma(q, i\omega_m) \\
\omega_m = 2\pi m T
\]
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\[ \Gamma^{-1}(q, i\omega_m) = \frac{1}{g} + \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} \left[ \tanh \left( \frac{\xi_+ - h}{2T} \right) + \tanh \left( \frac{\xi_- + h}{2T} \right) \right] \frac{1}{i\omega_m - \xi_- - \xi_+} \]

\[ \xi_{\pm} = \xi_p \pm q/2 \]
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\[
n = n_0(\mu, T) + 2 \int \frac{d\omega}{2\pi} \int \frac{d^3 q}{(2\pi)^3} n_B(\omega) \frac{\partial \delta(q, \omega)}{\partial \mu}
\]

\[
\delta = \text{Im } \log \Gamma
\]
\frac{\partial \delta(q, \omega)}{\partial \mu} \quad \text{Bound state}
\[
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\]
\frac{\partial \delta(q, \omega)}{\partial \mu}
\frac{\partial \delta(q, \omega)}{\partial \mu} = n_B(\omega)
\[ \frac{\partial \delta(q, \omega)}{\partial \mu} \]

\[ k_Fa \rightarrow -\infty \]

\[ T(M/N) = T_{\text{BEC}}(N) \left[ 1 - \frac{M}{N} \right]^{2/3} \]
Nozières and Schmitt-Rink 1985, Sá de Melo et al. 1993
16 A surprise

\[ n = n_0(\mu, h, T) + 2 \int \frac{d\omega}{2\pi} \int \frac{d^3q}{(2\pi)^3} n_B(\omega) \frac{\partial\delta(q, \omega)}{\partial\mu} \]

\[ m = m_0(\mu, h, T) + 2 \int \frac{d\omega}{2\pi} \int \frac{d^3q}{(2\pi)^3} n_B(\omega) \frac{\partial\delta(q, \omega)}{\partial h} \]
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Compressibility \(-\partial^2 F/\partial \mu_s \partial \mu_{s'}, s, s' = \uparrow, \downarrow\) NOT +ve semi-definite
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NSR treatment unphysical in crossover regime!
17 The two-channel model

\[ H = \sum_{p,s} \epsilon_p a^\dagger_{s,p} a_{s,p} + \sum_{q} \left( \frac{\epsilon_q}{2} + \varepsilon_0 \right) b^\dagger_q b_q + \frac{g}{\sqrt{V}} \sum_{p,q} b_q a^\dagger_{q+p,\uparrow} a^\dagger_{q+p,\downarrow} - p + \text{h.c.} \]

Feshbach boson
17 The two-channel model

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Feshbach boson

Scattering amplitude

\[ f(E) = -\frac{\hbar \gamma}{\sqrt{m}} \frac{1}{E - \varepsilon_0 + i\gamma \sqrt{E}} \]

with \( \gamma = \frac{g^2 m^{3/2}}{4\pi} \)
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Scattering length \( (f(0) = -a) \)

\[ a = -\frac{\hbar \gamma}{\sqrt{m} \varepsilon_0} \frac{1}{\sqrt{m} \varepsilon_0} \]
18 One channel or two?

Experimentally, resonances are broad \((\text{width } \gg \text{ other scales})\)
Exponentially, resonances are broad (width $\gg$ other scales)

For $\gamma \to \infty$ the two models are equivalent:

- At the two-particle level
Experimentally, resonances are broad (width $\gg$ other scales)

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- In mean field
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- With gaussian fluctuations
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...and presumably generally!
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- At the two-particle level
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- With gaussian fluctuations

...and presumably generally!

But for general \(\gamma\)

\[
\Gamma_{1C}^{-1}(q, i\omega_m) \rightarrow \frac{q^2}{4m} - i\omega_m + \tilde{\gamma}\Gamma_{1C}^{-1}(q, i\omega_m)
\]

\(\tilde{\gamma} \sim \gamma/\sqrt{E_F}\) is small parameter
19 The MIT experiment

Zwierlein et al. 2005
The Rice experiment

Partridge et al. 2005
Evidence for $P_{\text{crit}}$
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- Is it just thermal activation? $m_S(T \neq 0) \neq 0$
Evidence for $P_{\text{crit}}$

- Is it just thermal activation? $m_S(T \neq 0) \neq 0$
- Or fundamental change in g.s.? [strong qp interactions?]
• Quantum phase transitions \((T = 0)\)

Simplest approximation gave...
Quantum phase transitions \((T = 0)\) are more likely...
- Quantum phase transitions $(T = 0)$

more likely…

$m/n$

$1$

$N$

$PS$

$S$

$1/k_F a$

$2^{nd}$ order QPT
Quantum phase transitions ($T = 0$) more likely...

Boson-fermion coupling destroys condensation at $T = 0$ only a few examples known...
21 More structure...
More structure...

- Inhomogenous superfluidity

\[ \langle \psi_\downarrow(\mathbf{R} + \mathbf{r}/2)\psi_\uparrow(\mathbf{R} - \mathbf{r}/2) \rangle \propto e^{i\mathbf{Q} \cdot \mathbf{R}} F(\mathbf{r}) \]

\[ k_{F\uparrow} - k_{F\downarrow} \sim Q \]
21 More structure...

- Inhomogenous superfluidity

\[ \langle \psi_{\downarrow}(\mathbf{R} + \mathbf{r}/2)\psi_{\uparrow}(\mathbf{R} - \mathbf{r}/2) \rangle \propto e^{iQ \cdot \mathbf{R} F(\mathbf{r})} \]

\[ k_{F\uparrow} - k_{F\downarrow} \sim Q \]

FFLO
22 Summary

- Better understanding of two component fermi gas
  \[ N_{\uparrow} \neq N_{\downarrow} \]
  \[ \sim \text{Line of tricritical points in } (1/k_F a, m, T) \]
  \[ \sim \text{Atomic gases demand it!} \]
- Rich structure when \[ N_{\uparrow} \neq N_{\downarrow} \]
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- Analytic approaches to the broad resonance (single channel) limit DO require new ideas
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- Better understanding of two component fermi gas
  $\Rightarrow$ Atomic gases demand it!

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Detailed experimental phase diagram not far away!
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  \[
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- At tricritical point \( d_{\text{upper}} = 3 \)

\( \sim \text{only logarithmic corrections} \)
24 The ‘magnetized superfluid’ as a Fermi Liquid
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Effective Interaction of He$^3$ Atoms in Dilute Solutions of He$^3$ in He$^4$ at Low Temperatures

J. Bardeen, G. Baym,* and D. Pines
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- Phonon-mediated attraction between qp’s

  ~\rightarrow Possibility of [p-wave] pairing

- $p \cdot v_s$ coupling between qp’s and condensate
25 p-wave pairing of quasiparticles
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Superfluid phonons give attractive interaction

\[ U_{\text{ind}}(q, \omega) = U_{FB}^2 \chi(q, \omega) \]

\[ \chi(q, \omega) = \frac{n_B q^2}{m_B (\omega^2 - \omega_q^2)} \]

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Maximal p-wave gap

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Weak coupling only – in general need FL params
26 Generalized FFLO phases
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Add a qp with $p_0$, $\varepsilon^{(0)}(p_0)$ to moving fluid

$$E = \varepsilon^{(0)}(p_0) + p_0 \cdot v + \frac{1}{2} (M + m) v^2 \quad P = p_0 + (M + m) v$$
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qp energy and momentum in moving fluid

$$\varepsilon(p, v) = \varepsilon^{(0)}(p_0) + p_0 \cdot v + \frac{1}{2} m v^2,$$  
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qp energy and momentum in moving fluid

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\varepsilon(\mathbf{p}, \mathbf{v}) = \varepsilon^{(0)}(\mathbf{p}_0) + \mathbf{p}_0 \cdot \mathbf{v} + \frac{1}{2} m \mathbf{v}^2, \quad \mathbf{p} = \mathbf{p}_0 + m \mathbf{v}
\]

or in terms of \( \mathbf{p} \)

\[
\varepsilon(\mathbf{p}, \mathbf{v}) = \varepsilon^{(0)}(\mathbf{p} - m \mathbf{v}) + \mathbf{p} \cdot \mathbf{v} - \frac{1}{2} m \mathbf{v}^2.
\]
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\( f \) is fraction of dynamical effective mass \( m_d \) coming from dragging background superfluid.
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f_s = \frac{\rho_B - f \rho_F}{\rho_B + \rho_F}
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\( f_s < 0 \sim \text{FFLO?} \)
27 Outlook: phases of atomic matter

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Electric dipole moment
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    Electric dipole moment

- Lattice systems ...
Outlook II: quantum matter wave optics
28 Outlook II: quantum matter wave optics

$k_1 \quad k_2$

$k_1 \quad k_2$

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28 Outlook II: quantum matter wave optics

\[ k_1 \quad k_2 \]

\[ b = -1 \quad b = +1 \]

\[ E(p) \]

\[ \frac{2\pi}{T} \]