Journal for Person-Oriented Research

2015, 1(1-2), 99-109
Published by the *Scandinavian Society for Person-Oriented Research*
Freely available at http://www.person-research.org
DOI: 10.17505/jpor.2015.10

Adaptive Equilibrium Regulation: A Balancing Act in Two Timescales

Steven M. Boker

Department of Psychology
The University of Virginia
Charlottesville, VA 22903, USA

Email address:

boker@virginia.edu

To cite this article:

Boker, S. M. (2015). Adaptive equilibrium regulation: A balancing act in two timescales. *Journal for Person-Oriented Research*, 1, (1–2), 99–109. DOI: 10.17505/jpor.2015.10

Abstract: An equilibrium involves a balancing of forces. Just as one maintains upright posture in standing or walking, many self-regulatory and interpersonal behaviors can be framed as a balancing act between an ever changing environment and within-person processes. The emerging balance between person and environment, the equilibria, are dynamic and adaptive in response to development and learning. A distinction is made between equilibrium achieved solely due to a short timescale balancing of forces and a longer timescale *preferred equilibrium* which we define as a state towards which the system slowly adapts. Together, these are developed into a framework that this article calls Adaptive Equilibrium Regulation (ÆR), which separates a regulatory process into two timescales: a faster regulation that automatically balances forces and a slower timescale adaptation process that reconfigures the fast regulation so as to move the system towards its preferred equilibrium when an environmental force persists over the longer timescale. This way of thinking leads to novel models for the interplay between multiple timescales of behavior, learning, and development.

Keywords: dynamical systems analysis, multi-timescale analysis, self-regulation, person-oriented analysis

Introduction

Many metaphors have been employed to translate psychological, physiological, developmental, and medical theories into physical terms in order to enhance intuitive understanding. Momentum (Iso-Ahola & Mobily, 1980), spring and mass systems (Turvey, 1990), pendulums (Boker & Laurenceau, 2007; Reed, Barnard, & Butler, in press), thermostats (Chow, Ram, Boker, Fujita, & Clore, 2005), reservoirs (Deboeck & Bergeman, 2013) and elasticity (Boker, Montpetit, Hunter, & Bergeman, 2010) have been explored and sometimes empirically tested as models for processes that evolve over short (i.e., seconds), medium (i.e., days), or long (i.e., years) intervals of time. The appeal of these metaphors is that they can literally be embodied in order to provide a seed of understanding. For instance, every time we walk, we swing our legs and arms in ways that incorporate the actions of pendulums connected with variably elastic springs. Decades of self-locomotive experience allows us to deeply understand how a pendulum or mass and spring system will behave without the need to integrate systems of differential equations.

The physical metaphors mentioned above describe theories of *individual* action, adaptation and change. A single individual's developmental trajectory, self-regulation, or psychological states and traits thus must be studied as *processes* (Nesselroade, 1991) where the parameters of individuals' processes are first estimated and only then are population distribution characteristics derived (Molenaar, 2004). If research into human behavior and/or the aetiology of diseases is to result in theories that can be applied to individuals, this type of process-oriented research is essential (Boker, Molenaar, & Nesselroade, 2009).

The idea of behavioral processes evolving over time has been explored within the general framework of dynamical systems using a wide variety of contexts and models (e.g., Kelso, 1995; Pozzo, Levik, & Berthoz, 1995; Zanone & Kelso, 1992). This way of thinking about individual processes has led to interesting insights such as the idea of a *behavioral attractor*. An attractor describes the behavior of a process when it is in the vicinity of its *equilibrium*. Attractors and equilibria can be slippery concepts, so simple examples will be provided prior to introduction of a more rigorous mathematical section with statistical models that can be fit to data.

The current article develops a multi-timescale framework to model processes in terms of balancing forces: *Adaptive Equilibrium Regulation* (ÆR). This framework accounts for short term regulation as a separate process from longer term adaptation. This separation of regulation into models at two different timescales is useful since we can then map it to a wide range of phenomena such as perceptual learning and its underlying neural mechanisms (Schiltz et al., 1999), acquired tolerance and withdrawl symptoms in substance dependence (Tiffany, 1990), the honeymoon effect in romantic relationships (Aron, Norman, Aron, McKenna, & Heyman, 2000), or resiliency in daily positive affect (Boker, Montpetit, et al., 2010). The adaptive equilibrium regulation framework can be used to model the macro behavior of basic neural mechansims and as such is likely to be useful for the analysis of many systems that regulate sensitivity, expressivity, and activity in living organisms.

Concepts will be introduced from dynamical systems, differential equations, and introductory physics—the notions of force, acceleration, and equilibrium are central to translation of process-oriented theories into empirically testable models. Along the way, physical and psychological examples will be given in order to ground these relatively abstract notions in familiar territory. We will discuss errors likely to be made when typical analyses are used to model phenomena that exhibit adaptive equilibrium regulation. We suggest that while theories about average equilibria over populations may be well-served by commonly used methods, process-oriented and person-specific theories tested with population-oriented methods are likely to produce a variety of systematic erroneous conclusions. The article concludes that there are likely to be errors in the psychological, developmental, and medical literature that will continue to persist until person-oriented methods are used to test process-oriented theories.

What Are Equilibria?

When asked to "maintain your equilibrium", the first thought that comes to mind is likely to concern the act of upright standing. This is actually quite a complicated equilibrium since we can use flexion and tension at our ankles, knees, pelvis, spine, shoulders, elbows and wrists to manage this task and thus a large number of degrees of freedom are at our disposal. The task of upright standing is an example of an *unstable equilibrium*. That is to say, if we do not continuously and appropriately apply muscle flexion and tension, we will fall down. When motion tracked, it becomes obvious that people do not stand absolutely still when asked to maintain their equilibrium in upright stance, even when also instructed to "stand as still as you can" (Stoffregen, 1986). This continuous postural adjustment, termed *rambling and trembling* by Newell and colleagues (Slobounov, Moss, Slobounov, & Newell, 1998) can be seen as having at least three sources. First, small errors in compensation towards the unstable equilibrium point may cause over- or under-shoot and then must in turn be adjusted. In an ideal system, an unstable point equilibrium is infinitely small and so in real systems, one is unlikely to exactly achieve the equilibrium point. Second, one must take perception into account. Joint angles are perceived using stretch receptors that require some movement in order to signal (Clark, Horch, Bach, & Larson, 1979). Movement is also required for the inner ear and vision to contribute to adaptive compensating flexion and tension of the body (Berthoz, Lacour, Soechting, & Vidal, 1979; Stoffregen & Pittenger, 1995). As movement is reduced, the accuracy of perception of one's position relative to equilibrium is also reduced. Thus, it is adaptive to move when maintaining equilibrium in upright standing whereas attempting to stand completely motionless leads to inaccurate regulation of posture. Finally, the third reason one may move is that the environment may be changing. Standing on a sidewalk is different than standing on a train or tram. These three sources of variation in observed trajectories—equilibrium type, perception/regulation, and changing environment—form the basis of the adaptive equilibrium regulation framework.

Imagine, if you will, a wooden ball about 10 cm in diameter and painted red. Now imagine a circus performer giving the ball to a trained seal so that it can balance it on the tip of its nose. We applaud because we know that this is not an easy task. Even when the ball is in perfect equilibrium as in Figure 1-a, any small perturbation, perhaps a little gust of wind as in Figure 1-c, will cause the ball to fall unless the seal adapts to this changing context. In fact, our imaginary circus seal will be moving all the time, compensating for movements of the ball. The type of equilibrium is an *unstable point equilibrium* and so it is likely for the seal to under- or over-shoot. The seal is perceiving the ball's position and movement and then regulating by moving its nose in order to push the ball towards an infinitesimally small point equilibrium. Our imaginary small gusts of wind provide the changing environment. The clever seal does not drop the ball and we give it an ovation because we know the difficulty of this task.

The Oxford English Dictionary primary definition for equilibrium is "state of balance" (*The Concise Oxford Dictionary of the English Language*, 1982, pg. 326). But what is being balanced? Our clever seal could be said to be balancing the red wooden ball, but the ball is always in motion. What exactly is being balanced here? In essence, it is a balancing of forces: gravity versus the seal's nose. As we recall from introductory physics, force is mass times acceleration. The acceleration due to gravity is constant and in a constant direction from the seal's perspective. The mass in this system is the mass of the red wooden ball and that is also a constant. So the force applied by the seal must be equal in magnitude and opposite in direction to the force of gravity for the ball to continue to stay "in balance". The quotes are placed around "in bal-

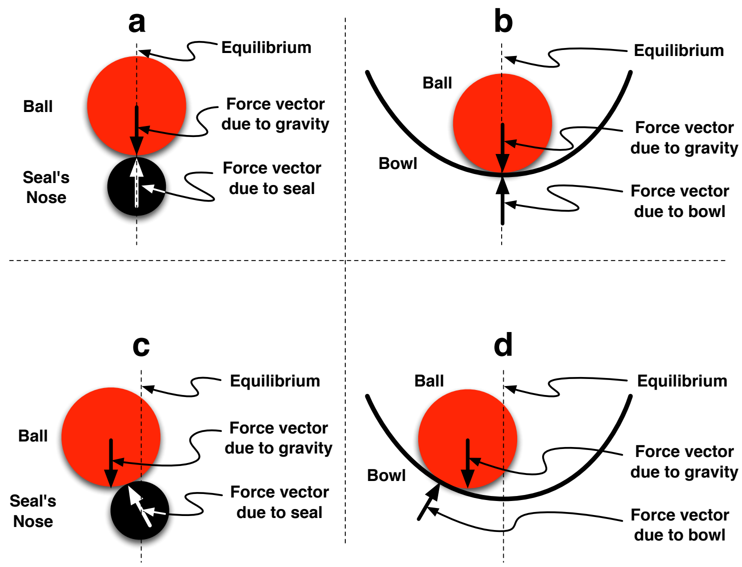

Figure 1: Graphic depictions of stable and unstable equilibria. a) The ball is placed exactly at the unstable equilibrium at the top of the seal's nose and the resulting force vectors exactly cancel. b) The ball is placed exactly at the stable equilibrium point at the bottom of the bowl and the forces vectors cancel. c) The ball is to the left of center of the seal's nose and the force vector from the seal's nose is orthogonal to the point of contact with the ball and so pushes the ball away from the equilibrium point, an example of an unstable equilibrium. d) The ball is to the left of the bottom of the bowl and the force vector from the bowl is orthogonal to the point of contact and so pushes the ball back towards the equilibrium point. Thus this is a stable equilibrium.

ance" because even though in aggregate the ball is being balanced, if we were to measure the vector of force applied to the ball by the nose of the seal at any instant in time, there is a vanishingly small probability that it would be exactly equal to -1 times the mass of the ball times the constant acceleration of gravity. However, if we take the mean of the force vectors applied by the seal's nose over the few minutes of the act, we will find that in the aggregate they do in fact balance the force exerted by gravity on the ball. While this is a gratifying result, note that performing the aggregation obscures the process by which the seal adapts to its perception of the ball so as to keep the ball from falling. This is an important distinction and one which often arises in research into human processes: aggregation helps to find *where* balance happens, but tells little or nothing about *how* that balance came about.

Now imagine that the red wooden ball is resting in a large mixing bowl with a rounded bottom as in Figure 1-b. If the ball is at the center of the bowl, it is at the bottom of the bowl. The upward force vector of the bowl exactly cancels the downward force vector produced by gravity and so the ball is "in balance" and can be said to be at its equilibrium. But now, if the ball is pushed away from the bottom of the bowl, the force vector produced by contact with bowl is pointed back towards the equilibrium as in Figure 1-d. This is called a *stable point equilibrium* and the ball will tend to roll back towards the equilibrium point no matter which direction it is pushed. This balancing act is too easy and so we do not applaud for the bowl. However, just as with the seal, if we take the mean of many measurements of the position of the ball as it is pushed by small gusts of wind, we can calculate an estimate of the point equilibrium.

It may seem trivial that the ball can be balanced by the bowl whereas it is much more difficult for the seal. But each of these toy examples are balancing acts. By decomposing these examples into questions about how and why the forces are being produced, we can better understand the processes involved — not just that there *is* balance, but *how* the balance is achieved. In order to estimate these forces, we will need to make repeated observations of the ball's position. Then, by using the information about how the ball's position changes, we can answer useful questions about the systems such as, "How is stability related to the shape of the bowl?" Or, "How is instablility related to the relative sizes of the ball and the seal's nose?" More deeply, with several different bowls and balls, we can even estimate the acceleration due to gravity and thus tell which planet we are on!

What data are required in order to estimate properties of the ball-and-bowl system? If we make measurements of the horizontal position of the ball every millisecond for an hour (60,000 position samples!), that would seem to be a rich source of data. Surely we wouldn't need any more than that. But suppose that the ball is just resting at the bottom of the bowl for the entire hour we are measuring. All of the measurements would be exactly the same number. The mean would tell us the position of the equilibrium. But, we would have no information about the shape of the bowl. In fact, we couldn't tell the difference between a bowl and a flat table if all of the measurements are the same. We need to observe the ball being

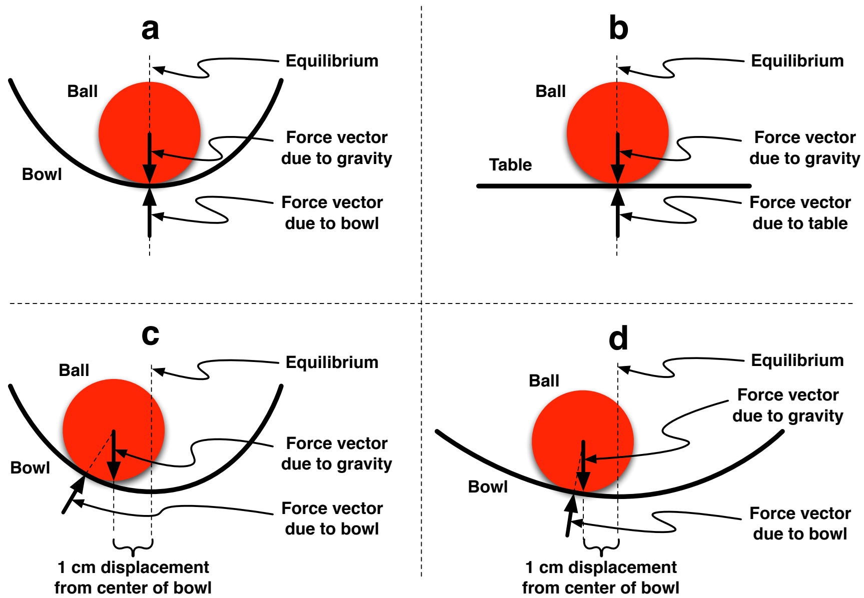

Figure 2: Graphic depictions of balanced and unbalanced forces. a) The ball is placed at the stable equilibrium at the bottom of the bowl and the resulting force vectors exactly cancel. b) The ball is placed anywhere on a level table and the force vectors cancel. c) The ball is to the left of the bottom of the bowl and the force vectors do not cancel. The angle between the two vectors relative to the distance from the center tells us about the shape of the bowl. d) The ball is the same distance to the left of the bottom of a more shallow bowl and the angle between the force vector from the bowl and the force vector of gravity is smaller.

regulated in order to estimate parameters of regulation. Looking back at Figure 1-d, it is the *time evolution* of the differences in the force vectors that tell us about the bowl. We can only estimate the force vectors when the system is not at equilibrium — when it is out of balance.

To see why we need imbalance in order to understand regulation, consider Figures 2-a and 2-b. When the ball is at equilibrium, either at the bottom of the bowl or sitting on a level table, the forces exactly cancel out. Since we can only measure the position of the ball and not directly measure the forces, there is no way to tell if we are on the earth or the moon. On the moon the force vectors would cancel each other exactly like they do here on earth and the ball would not move. Now consider the difference between Figures 2-c and 2-d. The ball is not at equilibrium and the force vectors do not cancel. For a given displacement from equilibrium, the steeper bowl has a larger mismatch between the force vector from the bowl and the force vector from gravity than does the more shallow bowl. This mismatch can be represented as an angle between the force vectors.

So, how does that help us? Remember that force is equal to mass times acceleration. Since the mass of the ball is constant, the difference between the two forces can be observed as acceleration of the ball. If we have many measurements of the position of the ball, we can estimate the second derivative (acceleration) of this time series for every measurement and thus estimate the mismatch in forces for each observed displacement from equilibrium. There are R functions such as GLLA (Boker, Deboeck, Edler, & Keel, 2010a) and GOLD (Deboeck, 2010) that give estimates of displacement, first derivative, and second derivative of time series. Derivative estimates of variables with respect to time are the pathway to understanding processes because these estimates directly measure the mismatch in forces in a system at each moment in time. When regularities in this mismatch are observed, parameters of regulatory processes can be estimated.

Of course, the physical systems discussed above are simple and the answer to "Which planet are we on?" should be obvious. But now consider psychological systems such as regulation of affect in the presence of stress and its relation in turn to cognitive performance. It is not obvious even what system of rules should apply. And that is what we are after — systematic rules that have application to systems of individual human behavior, development, and health. Thinking about these systems from the standpoint of equilibria and the forces that are operating is appealing since we can map the concept of physical force onto the concepts underlying structural equation modeling. A one unit change in a predictor variable produces some predicted change in an outcome variable —Increase the force that is supplied by one variable in a model and the imbalance of forces is resolved into a new equilibrium.

When we make intuitive appeals to physical analogies such as "mood swings" or "under pressure" or "resilience" are we dragging along physical baggage like "momentum", "heat" or "elasticity" inappropriately? Or do these metaphors actually make sense? One thing is clear: aggregation methods that estimate only the means and variances of variables across individuals are unable to test

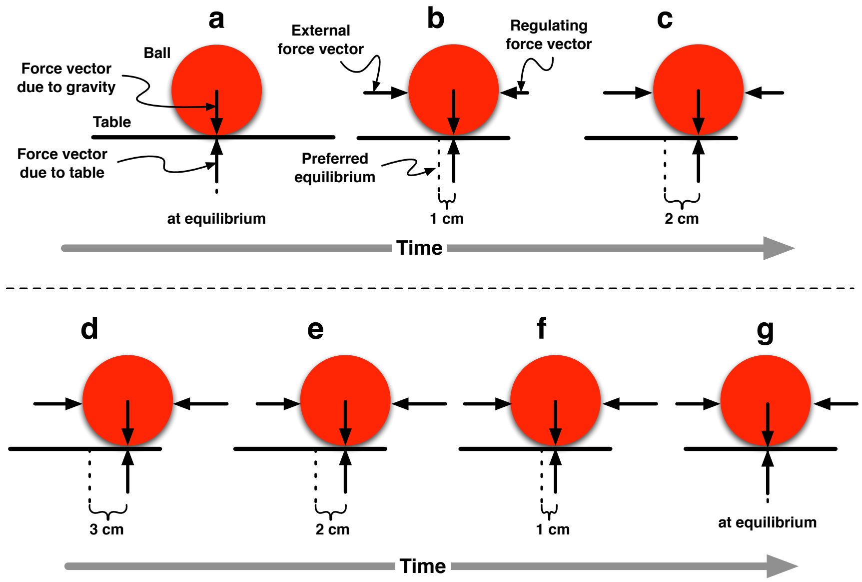

Figure 3: Graphic depictions of regulatory forces. a) The ball is placed at a preferred equilibrium on a level table. b) An external force is applied which moves the ball to the right and a regulatory force is generated due to the 1cm discrepency between the current and preferred position. c) As the position becomes more discrepant, the regulating force becomes larger until, d) the regulatory force exceeds the external force and e) the ball begins to move back towards the preferred equilibrium. f) The regulating force reduces as the ball approaches equilibrium until f) the forces are balanced exactly at equilibrium.

these sorts of process-oriented metaphors because they only estimate values for the equilibria and not the processes by which mismatches in forces are resolved into equilibria. We will need to understand relationships between variables in terms of effects that have properties similar to those exhibited by force in physics if we are to understand and test models of processes.

What is Regulation with Respect to Equilibrium?

The adaptive equilibrium regulation framework for process-oriented models includes three components: equilibrium type, perception/regulation, and environmental context. So far, we have examined some consequences of two categories of equilibria: stable and unstable. There is a third interesting category of equilibria — combinations of both stable and unstable properties in the same equilibrium. This third category of equilibria is the one that includes the possibility of chaotic processes. In the remainder of the article we will consider a set of important regulatory systems with equilibria that are either only stable or only unstable.

How do we know if a system is not currently in balance and how do perception and regulation lead to the formation of a balanced state? As an example, consider a psychological variable such as positive affect (Watson, Clark, & Tellegan, 1988). When measured intensively over time within individual, self-reported positive affect tends to fluctuate with a relatively short interval timescale (e.g. Steele & Ferrer, 2011; Erbacher, Schmidt, Boker, & Bergeman, 2012; Ong, Bergeman, Bisconti, & Wallace, 2006). The within-person mean of positive affect is a reasonable estimator of the central tendency of the distribution of within-person scores. Thus, it seems not unreasonable to hypothesize that there exists a process that regulates positive affect and that process may operate in such a way that an equilibrium will be observed. This appeals to intuition, since one often hears individuals report being "out of balance" with respect to affect. While we would not wish to accept an appeal to intuition as empirical evidence, we can at least say that it is plausible that individuals can perceive their own momentary affective state relative to their preferred equilbrium.

Given a perception of imbalance in positive affect, how would regulation work? Let us consider this process-oriented system as a thought experiment in terms of balancing of forces. In our thought experiment we will assume that there exists a preferred equilibrium, that is to say a level of positive affect at which the person functions best. We will then consider how regulation over the short term and adaptation over the long term might come about.

Suppose our red wooden ball is resting on a table at its preferred equilibrium as shown in Figure 3-a. Next, suppose that a force from the environment pushes the ball towards the right as in Figure 3-b. We will suppose that this force is constant during our thought experiment. The ball can perceive that its position is being displaced to the right and so a regulating force appears in Figure 3-b pushing to the left. But the regulating force is too small, so the ball's position continues to be displaced further to the right in Figure 3-c. The regulating force grows until it is larger than the external force in Figure 3-d and the ball begins to move back to the left. As the ball approaches its preferred equilibrium, the regulating force reduces until it balances the external force so that in Figure 3-g the ball has returned to its preferred equilibrium and all the forces are balanced.

What does the wooden ball need to know in order to perform this regulation task? First, it needs to be able to estimate its displacement from its preferred equilibrium. In this way it can increase the regulating force until it exceeds the external force. But suppose displacement was all it knew. As the ball approached equilibrium, the regulating force would continue to be reduced until it became smaller than the external force and the ball would again move away from equilibrium. If all the ball knows is its distance from equilibrium, we will find that this system will oscillate indefinitely and always be to the right of the preferred equilibrium. In order for the appropriate regulating force to be found that exactly balances the external force (as in Figure 3-g), the ball must have a way of estimating the magnitude of the external force. This means that it must be able to estimate not only how far it has been displaced from its equilibrium, but also whether the currently supplied regulating force is sufficient to move the ball back towards its equilibrium. Thus it needs to know both its position and velocity relative to equilibrium — if at a given displacement from equilibrium its velocity is *away* from equilibrium it needs to *increase* the regulating force, but at the same displacement if the velocity is *towards* equilibrium it needs to *decrease* the regulating force.

Let us return to the psychological example of positive affect. Suppose some positive influence, perhaps a new friend, pushes an individual's affect above the personal baseline equilibrium for positive affect. Over time, suppose that this continuing positive influence is regulated such that the individual returns to his or her personal baseline. Now, suppose that the positive influence is removed — the friend moves to a distant town. This model for affective regulation would predict that there would be something like a rebound effect such that the individual's positive affect would be automatically decreased due to the removal of the external force. The post-friendship depression would increase until the regulating system can perceive that the individual's positive affect is too low, and then the internal regulating force causing the depression would decrease until it changed sign and pushed positive affect back up to its baseline equilibrium. While a thought experiment is insufficient evidence to support a theory, we can say that this model of regulation is not inconsistent with intuitive experience.

This adaptive equilibrium regulation model is one way of thinking about dependency. Removing a positive force produces change in affect in the negative direction. This type of balancing of forces model could be applied to a wide variety of psychological and health systems, e.g., romantic couples or substance use. The period of higher positive affect just after application of the positive environmental force maps to what people call the "honeymoon" period. The depression just after removal of the positive environmental force maps to "withdrawl". For example, when an reformed alcoholic starts drinking again, there is a period of exhilaration before drinking becomes the norm. Tolerance builds up as larger amounts of alcohol are needed to obtain the "honeymoon" effect. Eventually, drinking is maintained solely to keep withdrawl symptoms at bay. This substance dependency example maps to the regulation model depicted in Figure 3.

The same model predictions could be applied to a negative force. At first the negative force depresses positive affect, but eventually equilibrium is achieved back at the preferred state. When the negative force is removed, a rebound is predicted to occur. This rebound effect could be summarized using the colloquial adage, "Why am I beating my head on the wall? Because it feels so good when I stop."

Now let's return to the red wooden ball in the bowl as in Figure 4-a. How does this balancing-of-forces model for regulation relate to how the ball rolls around in the bowl? Suppose that the preferred equilibrium point for the ball is at the center of the bowl. With no external forces and if the bowl is level and symmetric, the bottom of the bowl exactly coincides with the center of the bowl and so the ball is at a stable and preferred equilibrium. If a constantly applied external force pushes the ball to the left, a regulating force in the opposite direction is automatically generated due to the curvature of the bowl as in Figure 4-b. The larger the displacement from equilibrium is, the larger the regulating force will be. But this results in the situation shown in Figure 4-c where there is a balancing of forces such that the ball comes to equilibrium to the left of the preferred equilibrium at the center of the bowl. Now by tipping the bowl, as in Figure 4-d, the regulating force vector is increased and the ball moves back towards equilibrium until in Figure 4-e balancing of forces is achieved and the center of the ball is directly over the center of the bowl. But it is evident that if the external force is removed, the ball will move away from the center of the bowl until the bowl can be retilted to again be level.

The ball-and-bowl model is one way to implement the regulation of positive affect example. The mechanism that tilts the bowl will need to know the displacement from preferred equilibrium and the velocity of the ball in order to appropriately adapt the tilt of the bowl. The argument is the same as it was for the affect regulation model: for a given displacement from equilibrium, if the ball is moving *away* from the center, the bowl needs to be *tilted more* but if the ball is moving *towards* the center, the bowl needs to be *tilted less*. We find this to be a useful construct because it separates the part of the regulation due to the shape of the bowl, i.e., the part that is dependent only on displacement from equilibrium, from the part that attends to a combination of velocity and displacement, i.e., the bowl tilting mechanism.

Although the bowl itself forms an attractor around a stable point equilibrium, once a mechanism is invoked to tilt the bowl, the system could be considered to be *quasi-stable*. That is to say, even if the bowl is in balance at the equilibrium point as in Figure 4-e, removal of an external force will cause the ball to move away from equilibrium. This is not an unstable equilibrium since even if the tilt-

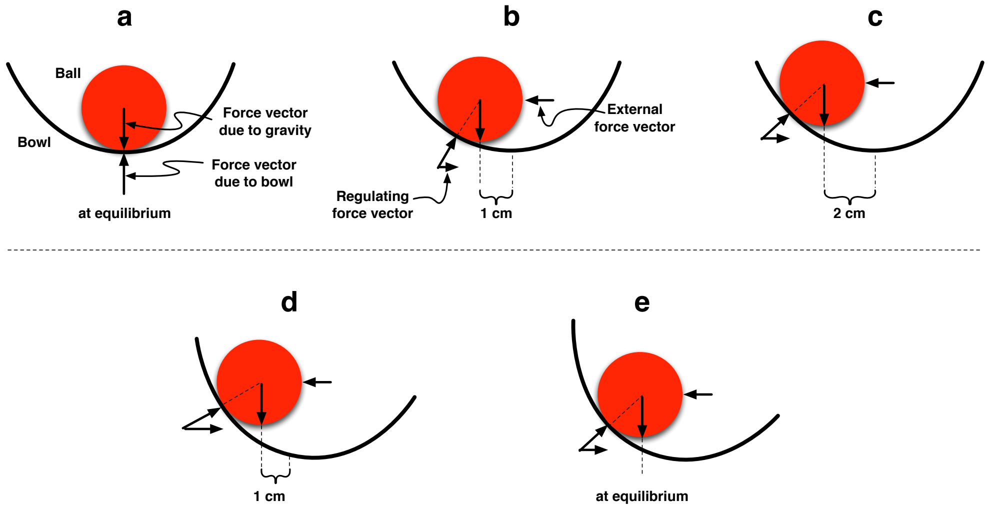

Figure 4: Graphic depictions of regulatory forces acting in a ball-and-bowl model. a) The ball is placed at a preferred equilibrium at the center of the bowl and forces are balanced since the center of the bowl is also the bottom of the bowl. b) An external force is applied which moves the ball to the left and a regulating force is generated since the 1 cm discrepency is related to the curvature of the bowl. However, the regulating force is to small and so c) the ball moves up the side of the bowl until it stops when the forces balance at 2 cm from its preferred equilibrium point at the center of the bowl. d) Since there still is a discrepency between the center of the bowl and the current position, the bowl is tipped until the regulating force exceeds the external force. e) the bowl is tipped until the forces are balanced and the position is at the preferred equilibrium at the center of the bowl.

ing mechanism breaks, the ball will still come into balance somewhere in the bowl. However, this is an example of a non-ergodic system. Since the bowl is essentially reconfiguring itself to respond to changing context in the environment, we cannot rely on a randomly selected time interval to be a representative sample to generate aggregate estimates for a population of bowls. The bowl is tilting itself based on the context of environmental forces around it, but the bowl does not need to change shape.

We can recover the shape of the bowl and the parameters of the tilting mechanism for each person by studying the *movement* of the ball for each person, i.e., a person-oriented time series. If the ball does not move, there is no information to estimate person-specific regulation and adaptation—only by studying *imbalance* do we learn about *balance*. A cross sectional study provides only a single observation per person and so provides no estimate of movement and therefore cannot tell us about regulation and adaptation. Estimating person-specific regulation and adaptation is the only way we can recover the nomothetic laws that govern regulation and adaptation in the population.

How might changes in environment lead to changes in regulation? A possible answer is that there exists a set of general principals that allow a regulating organism to balance forces to achieve equilibria. The resulting equilibria may themselves change as characteristics of the environment change and as the organism develops and ages. The equilibria and the parameters of regulating processes may be very different in childhood and in old age, but the principals of balancing forces could apply equally to any age. It is also important to recall that people make choices about and changes to their environments. One effective regulatory strategy is to change aspects of the environment: If one is cold, throw another log on the fire. Another strategy is to select a new environment: If one is cold while outside and there is no chance of affecting the environment, go indoors. Thus regulation can effect changes both in the regulating force as well as in the environmental force. Regulation effects flow in both directions: environment can influence equilibria and regulation, which in turn can influence the environment.

Identifying and Estimating Regulatory Processes.

In order to translate a theory that uses the adaptive equilibrium regulation framework into a testable model, the three components—equilibrium type, perception/regulation, and environmental context—must be mapped onto model equations. We have, through thought experiments, demonstrated that acceleration and velocity of the system are critical inputs to mechanisms that would instantiate adaptive equilibrium regulation. We have also noted that there are two timescales that must be taken into account. With these constraints in mind, it is evident that formulating the model in terms of differential equations will be a useful step.

In general, we have seen that imbalance in forces leads to a prediction of acceleration in the short term regulation process as a function, f of the difference between the force vectors acting on the system. So, for a variable, x, modeled at time, t,

$$\frac{d^2x}{dt^2}(t) = f(\Delta F) \tag{1}$$

where where $\frac{d^2x}{dt^2}(t)$ is the second derivative of x at time t and ΔF is the difference in the force vectors. In our physical examples, this regulation can be expressed as a function of displacement from the preferred equilibrium. Thus, if a variable x is modeled at time t with preferred equilibrium E_p then

$$\frac{d^2x}{dt^2}(t) = f(E_p - x(t)) \tag{2}$$

where $\frac{d^2x}{dt^2}(t)$ is the second derivative of x at time t and $f(E_p - x(t))$ is some function of the difference between the preferred equilibrium and the value of x at time t. Of course, it is possible that quantities other than just the displacement from equilibrium will be needed to fully specify the function f. One linear model that expresses acceleration in terms of a function of displacement from equilibrium is a linear oscillator, which in physics is the model for an ideal zero length spring or a frictionless pendulum,

$$\frac{d^2x}{dt^2}(t) = \eta(E_p - x(t)) \tag{3}$$

where here f is just a proportion $-1.0 >= \eta >= 1.0$ times the displacement from equilibrium. When η is negative, the equilibrium type for this model is a stable point equilibrium. This maps exactly to the ball-and-bowl described earlier when the bowl is parabolically shaped and the η represents the steepness of the bowl: if η is negative, the bowl creates a stable equilibrium at its lowest point and if η is positive, the bowl is upside down and represents an unstable equilibrium. Often, it is useful to introduce a friction term into this model, giving

$$\frac{d^2x}{dt^2}(t) = \eta(E_p - x(t)) + \zeta\frac{dx}{dt}(t) \tag{4}$$

where ζ is a linear coefficient expressing friction (if negative) or amplification (if positive) and $\frac{dx}{dt}(t)$ is the first derivative of x at time t. This is the damped linear oscillator (DLO) model that has been increasingly used in the past two decades to account for fluctuations in within-person time series such as daily affect (Pettersson, Boker, Watson, Clark, & Tellegen, 2013), coupling during physical coordination tasks (Butner, Amazeen, & Mulvey, 2005), and emotion regulation in widowhood (Bisconti, 2001) to name a few.

Our thought experiments led us to conclude that adaptation to a persistent force will require, at a minimum, knowing how far the system is from preferred equilibrium and also its first derivative. So, for a system where there is a constant external force F_e, we can model the adaptive force, F_a, as

$$\frac{dF_a}{dt}(t) \quad = \quad Mg((E_p - x(t)), \frac{dx}{dt}(t)) \tag{5}$$

where $\frac{dF_a}{dt}(t)$ is the first derivative of the adaptive force, M is a constant mass, and g is a function of the displacement from preferred equilibrium $(E_p - x(t))$ and $\frac{dx}{dt}(t)$ the current rate of change in x. Since all of the forces are acting with respect to the same constant mass M, mismatch in forces again can be expressed in terms of mismatch in accelerations by dividing both sides of Equation 5 by M. Thus the first derivative of the adaptive force can be expressed as a change in acceleration, i.e., the third derivative of the variable x with respect to time,

$$\frac{d^3x}{dt^3}(t) = g((E_p - x(t)), \frac{dx}{dt}(t)) \tag{6}$$

where $\frac{d^3x}{dt^3}(t)$ is the third derivative of x with respect to time at time t.

A simple linear model for this type of system maps onto the tilted bowl thought experiment such that the function g is a linear combination of the long term displacement and velocity that accounts for the rate of change in adaptive force. In order to keep this distinction between short and long timescales clear, we will denote the contribution of the long timescale displacement from equilibrium as $(E_p - x_L(t))$ and its associated velocity $\frac{dx_L}{dt}$. The linear model for long term adaptation can now be written as

$$\frac{d^3x_L}{dt^3}(t) = \alpha_1(E_p - x_L(t)) + \alpha_2\frac{dx_L}{dt}(t), \tag{7}$$

where α_1 is a proportional constant is the degree to which displacement from preferred equilibrium is avoided and α_2 represents the proportional constant for the ball's velocity. One can now map these parameters to a bowl tilting mechanism where the change in tilt (change in relative acceleration) increases when the displacement from preferred equilibrium increases and also increases when the ball is moving away from equilibrium. If this mechanism is operating at the same timescale as the short timescale regulation then the equilibrium type of the combined system might not be stable. However, when the timescale of the tilting mechanism is sufficiently larger than the timescale of regulation then the equilibrium of the combination of regulation and adaptation is stable whenever the short and long timescale equilibria are both stable.

The forces acting in the short timescale regulation must also include those forces due to long timescale adaptation. The total regulating force is thus some function of both the force provided by both regulation and adaptation. The simplest function for combining is a linear combination of these two forces as occurs in the ball-and-bowl model. The tilt of the bowl increases the angular diffence between gravity and bowl for a given distance from preferred equilibrium. By examining Equation 7 we see that if the ball is at the preferred equilibrium $(E_p - x_L(t) = 0)$ and the ball is not moving $(\frac{dx_L}{dt}(t) = 0)$ the tilt is not changing — the tilt is at equilibrium. As long as α_1 and α_2 are chosen appropriately, the tilt equilibrium is stable. Thus, if the tilt is perturbed away from equilibrium, it returns to equilibrium. At short timescales, if $E_p - x(t) = 0$ and $\frac{dx}{dt}(t) = 0$ the short timescale regulation mechanism in Equation 4 is also at a stable equilibrium. Thus, the system as a whole has a stable equilibrium point at the preferred equilibrium.

To find the acceleration contribution of the long timescale adaptation to the short term forces for a chosen time, t, we can take the specific integral of Equation 7 as

$$\frac{d^2x_L}{dt^2}(t) = C(t) + \alpha_2(E_p - x_L(t)) \quad (8)$$

where $C(t)$ is the constant of the specific integral. Note that for a chosen time t, the displacement term from Equation 7, $\alpha_1(E_p - x_L(t))$, is a constant and so is subsumed into the constant, $C(t)$. However, this term, $C(t)$, is not the same for all t. The adaptation mechanism provides a balancing acceleration, but as the tilt of the bowl changes with time, the acceleration provided by the bowl also changes with time.

When the system is at equilibrium, $E_p - x(t) = 0$ and so the acceleration due to the tilt of the bowl reduces to $C(t)$. In order to balance the forces at equilibrium, the adaptive force due to the tilt, $F_a = MC(t)$ must be counteract the persistent and constant external force, $F_e = MA_e$, where A_e is the external acceleration. Thus for a given mass M,

$$\begin{aligned} MC(t) &= MA_e & (9) \\ C(t) &= A_e & (10) \end{aligned}$$

Thus the specific integral constant, $C(t)$, is equal to the externally applied acceleration, A_e, when time t is chosen to be such that the system is at equilibrium. In this way, an adaptive equilibrium regulation model can estimate and compensate for environmental forces that persist over the longer timescale at which adaptation happens while allowing the short term regulation mechanism to accommodate and regulate momentary environment forces.

When fitting models such as these using filtering methods such as Generalized Local Linear Approximation (Boker, Deboeck, Edler, & Keel, 2010b) or Latent Differential Equations (Boker, Neale, & Rausch, 2004), the number of observations (or window) covered by the filter needs to be specified. This can be accomplished by choosing the number of columns for time delay embedding (Sauer, Yorke, & Casdagli, 1991; Oertzen & Boker, 2010) for fitting this part of the model. A smaller window results in the estimation of derivatives focusing on a shorter timescale and a wider window estimates for longer timescales. This can be used to separate the contribution of shorter and longer timescales. For instance a process is sampled once an hour and if a 5 column embedding is used for the shorter timescales and a 24 column embedding is used for the longer timescales, the shorter embedding will tend to focus on derivatives that exhibit change over an interval of 2 hours or so. On the other hand, the longer embedding will tend to smooth over those shorter intervals and focus on average changes over half a day or so. Both embeddings can be constructed side by side so that each row of a multi-timescale time delay embedded matrix includes columns for the shorter timescale and columns for the longer timescale.

The closer the shorter and longer timescales are to one another, the more likely the regulation and adaptation cannot be so simply separated. For instance, consider once again the circus seal balancing the ball on its nose. One reason that this task is so difficult is that the timescales of regulation and adaptation are so similar. Balancing a 2 meter pole on one finger is much easier than balancing a basketball on one finger (Foo, Kelso, & Guzman, 2000). The reason that the pole is easier is that the length of the pole increases the difference between timescales of regulation and adaptation. Regulation and adaptation to unstable equilibria become easier both for the regulating individual and for statistical estimation as timescales become separable.

Conclusions

As organisms regulate themselves and adapt to environments, mechanisms must be present to respond to momentary, i.e., short timescale, perturbations as well as longer term persistent forces. We have explored some thought experiments about how regulation and adaptation might be instantiated in human systems and developed a framework we called Adaptive Equilibrium Regulation (ÆR). The framework is plausible in that it could model commonly observed phenomena such as the "honeymoon effect" in romantic relationships, drug dependence and withdrawal, perceptual learning in vision, or resiliency in affect regulation. Models for regulation and adaptation were built using the logic that there must be a balancing of forces in order for a system to have an equilibrium. We then translated the thought experiments into testable differential equations models.

The ÆR framework is a useful construct in that it helps identify and clarify ways in which regulation and adaptation might be instantiated. However, at this point, this is purely a theoretic contribution. The proposed models must be tested using empirical person-oriented timeseries if they are to be truly helpful in understanding human systems. In addition, we have simplified our presentation to one-dimensional dynamics: models with just a single variable to be regulated. As more variables are introduced, or as coupled systems (such as a pair of romantically involved individuals) are modeled, the dynamics and the equations become ever more complex; regulating and adapting in ever more varied ways. If we are to study human processes, we must embrace this complexity in our person oriented models.

It is worthwhile to remember the somewhat surprising result that only by studying systems when they are out of balance can we understand how these systems regulate their balance. Only by studying systems that are changing can we understand how systems come to rest. This could be expressed as a paraphrase on the Chinese philosophy of wú wéi, the philosophy of "action from inaction". The ÆR philosophy might be stated as, "balance from imbalance".

Finally, it cannot be over emphasized that commonly used aggregation methods result in knowledge about the location of equilibria, but obscure the processes by which these equilibria are formed and maintained. Only by using intensive longitudinal measurement of individuals and modeling these timeseries can we understand the processes of regulation and adaptation. We need to first model how an individual regulates and how an individual adapts in order to estimate and understand individual differences in regulation and adaptation.

References

Aron, A., Norman, C. C., Aron, E. N., McKenna, C., & Heyman, R. E. (2000). Couples' shared participation in novel and arousing activities and experienced relationship quality. *Journal of Personality and Social Psychology*, *78*(2), 273.

Berthoz, A., Lacour, M., Soechting, J., & Vidal, P. (1979). The role of vision in the control of posture during linear motion. *Progressive Brain Research*, *50*, 197–209.

Bisconti, T. L. (2001). *Widowhood in later life: A dynamical systems approach to emotion regulation.* Unpublished doctoral dissertation, University of Notre Dame. (Unpublished doctoral dissertation)

Boker, S. M., Deboeck, P. R., Edler, C., & Keel, P. K. (2010a). Generalized local linear approximation of derivatives from time series. In S.-M. Chow, E. Ferrer, & F. Hsieh (Eds.), *Statistical methods for modeling human dynamics: An interdisciplinary dialogue* (pp. 161–178). Boca Raton, FL: Taylor & Francis.

Boker, S. M., Deboeck, P. R., Edler, C., & Keel, P. K. (2010b). Generalized local linear approximation of derivatives from time series. In S.-M. Chow & E. Ferrar (Eds.), *Statistical methods for modeling human dynamics: An interdisciplinary dialogue* (pp. 161–178). Boca Raton, FL: Taylor & Francis.

Boker, S. M., & Laurenceau, J. P. (2007). Coupled dynamics and mutually adaptive context. In T. D. Little, J. A. Bovaird, & N. A. Card (Eds.), *Modeling ecological and contextual effects in longitudinal studies of human development* (pp. 299–324). Mahwah, NJ: Lawrence Erlbaum Associates.

Boker, S. M., Molenaar, P. C. M., & Nesselroade, J. R. (2009). Issues in intraindividual variability: Individual differences in equilibria and dynamics over multiple time scales. *Psychology & Aging*, *24*(4), 858–862.

Boker, S. M., Montpetit, M. A., Hunter, M. D., & Bergeman, C. S. (2010). Modeling resilience with differential equations. In P. Molenaar & K. Newell (Eds.), *Learning and development: Individual pathways of change* (pp. 183–206). Washington, DC: American Psychological Association.

Boker, S. M., Neale, M. C., & Rausch, J. (2004). Latent differential equation modeling with multivariate multi-occasion indicators. In K. van Montfort, H. Oud, & A. Satorra (Eds.), *Recent developments on structural equation models: Theory and applications* (pp. 151–174). Dordrecht, Netherlands: Kluwer Academic Publishers.

Butner, J., Amazeen, P. G., & Mulvey, G. M. (2005). Multilevel modeling to two cyclical processes: Extending differential structural equation modeling to nonlinear coupled systems. *Psychological Methods*, *10*(2), 159–177.

Chow, S. M., Ram, N., Boker, S. M., Fujita, F., & Clore, G. (2005). Capturing weekly fluctuation in emotion using a latent differential structural approach. *Emotion*, *5*(2), 208–225.

Clark, F., Horch, K., Bach, S., & Larson, G. (1979). Contributions of cutaneous and joint receptors to static knee-position sense in man. *Journal of Neurophysiology*, *42*(3), 877–888.

The Concise Oxford Dictionary of the English Language. (1982). Oxford: Oxford University Press.

Deboeck, P. R. (2010). Estimating dynamical systems: Derivative estimation hints from Sir Ronald A. Fisher. *Multivariate Behavioral Research*, *45*(4), 725–745.

Deboeck, P. R., & Bergeman, C. (2013). The reservoir model: A differential equation model of psychological regulation. *Psychological methods*, *18*(2), 237.

Erbacher, M. K., Schmidt, K. M., Boker, S. M., & Bergeman, C. S. (2012). Measuring positive and negative affect in older adults over 56 days: Comparing trait level scoring methods using the partial credit model. *Applied Psychological Measurement*, *13*(2), 146–164.

Foo, P., Kelso, J. A. S., & Guzman, G. C. de. (2000). Functional stabilization of unstable fixed points: Human pole balancing using time-to-balance information. *Journal of Experimental Psychology: Human Perception and Performance*, *26*(4), 1281–1397.

Iso-Ahola, S. E., & Mobily, K. (1980). Psychological momentum: A phenomenon and an empirical (unobtrusive) validation of its influence in a competitive sport tournament. *Psychological Reports*, *46*(2), 391–401.

Kelso, J. A. S. (1995). *Dynamic patterns: The self–organization of brain and behavior.* Cambridge, MA: MIT Press.

Molenaar, P. C. M. (2004). A manifesto on psychology as idiographic science: Bringing the person back into scientific psychology, this time forever. *Measurement: Interdisciplinary Research and Perspectives*, *2*(4), 201–218.

Nesselroade, J. R. (1991). The warp and woof of the developmental fabric. In R. Downs, L. Liben, & D. S. Palermo (Eds.), *Visions of aesthetics, the environment, and development: The legacy of Joachim F. Wohlwill* (pp. 213–240). Hillsdale, NJ: Lawrence Erlbaum Associates.

Oertzen, T. v., & Boker, S. M. (2010). Time delay embedding increases estimation precision of models of intraindividual variability. *Psychometrika*, *75*(1), 158–175.

Ong, A., Bergeman, C. S., Bisconti, T. L., & Wallace, K. A. (2006). Psychological resilience, positive emotions, and successful adaptation to stress in later life. *Journal of Personality and Social Psychology*, *91*(4), 730–749.

Pettersson, E., Boker, S. M., Watson, D., Clark, L. A., & Tellegen, A. (2013). Modeling daily variation in the affective circumplex: A dynamical systems approach. *Journal of Research in Personality*, *47*(1), 57–69.

Pozzo, T., Levik, Y., & Berthoz, A. (1995). Head and trunk movements in the frontal plane during complex dynamic equilibrium tasks in humans. *Experimental Brain Research*, *106*, 327–338.

Reed, R. G., Barnard, K., & Butler, E. A. (in press). Distinguishing emotional co-regulation from codysregulation: An investigation of emotional dynamics and body-weight in romantic couples. *Emotion*.

Sauer, T., Yorke, J., & Casdagli, M. (1991). Embedology. *Journal of Statistical Physics*, *65*(3,4), 95–116.

Schiltz, C., Bodart, J., Dubois, S., Dejardin, S., Michel, C., Roucoux, A., et al. (1999). Neuronal mechanisms of perceptual learning: changes in human brain activity with training in orientation discrimination. *Neuroimage*, *9*(1), 46–62.

Slobounov, S. M., Moss, S. A., Slobounov, E. S., & Newell, K. M. (1998). Aging and time to instability in posture. *Journal of Gerontology: Biological Sciences*, *53A*, 871–878.

Steele, J., & Ferrer, E. (2011). Latent differential equation modeling of self-regulatory and coregulatory affective processes. *Multivariate Behavioral Research*, *46*, 994-1003.

Stoffregen, T. A. (1986). The role of optical velocity in the control of stance. *Perception & Psychophysics*, *39*(5), 355–360.

Stoffregen, T. A., & Pittenger, J. B. (1995). Human echolocation as a basic form of perception and action. *Ecological Psychology*, *7*(3), 181–216.

Tiffany, S. T. (1990). A cognitive model of drug urges and drug-use behavior: role of automatic and nonautomatic processes. *Psychological Review*, *97*(2), 147.

Turvey, M. T. (1990). Coordination. *American Psychologist*, *45*(8), 938–953.

Watson, D., Clark, L. A., & Tellegan, A. (1988). Development and validation of brief measures of positive and negative affect: The PANAS scales. *Journal of Personality and Social Psychology*, *54*(6), 1063–1070.

Zanone, P. G., & Kelso, J. A. S. (1992). Evolution of behavioral attractors with learning: Nonequilibrium phase transitions. *Journal of Experimental Psychology*, *18*, 403–421.