SHORT ACCESSIBLE PAPERS IN COMMUTATIVE ALGEBRA

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This is a personal and idiosyncratic list of some classic papers on commutative algebra that should be accessible after a first (sometimes second) course in Commutative Algebra (and, in some cases, a course on algebraic geometry.) We hope to update the list someday; we welcome suggestions and comments.

Organization. The papers are grouped by major subfield, and each one has one or more of the following tags, to better indicate its contents:
AG: = algebraic geometry
HA: = homological algebra
IT: = ideal theory
MT: = module theory
CP: = char p methods
GA: group actions
RT: ring theory
GB: Gröbner bases
MI: monomial ideals

The Papers

Ideal Theory.

- IT: Bass, Hyman: On the ubiquity of Gorenstein rings. Math. Z. 82 1963 828.
  A classic paper introducing Gorenstein rings as rings of finite injective dimension. Pick and choose from this paper.
- IT: Bruns, Winfried: The Eisenbud-Evans generalized principal ideal theorem and determinantal ideals. Proc. Amer. Math. Soc. 83 (1981), 19–24.
  Bounding the possible heights of ideals given in a structured way, such as determinantal ideals.
- IT: Burch, Lindsay: A note on the homology of ideals generated by three elements in local rings. Proc. Cambridge Philos. Soc. 64 1968 949–952.
  Proves that a regular ring modulo an ideal with just three generators can have aribitrary depth. For a far-reaching generalization see Bruns' paper ' "Jede" endliche freie Auflösung. . . ' below.

Date: January 20, 2011.

- IT: Cowsik, R. C.; Nori, M. V.: Affine curves in characteristic p are set theoretic complete intersections. Invent. Math. 45 (1978), no. 2, 111–114.
  The title says it all.
- IT: Cowsik, R. C.; Nori, M. V.: On the fibres of blowing up. J. Indian Math. Soc. (N.S.) 40 (1976), no. 1-4, 217–222 (1977).
  Relates the "analytic spread" of an ideal—that is, the dimension of the special fiber of the Rees algebra of the ideal—to the depth of the ring modulo powers of the ideal.
- IT, GB, HA: Derksen, Harm; Sidman, Jessica: A sharp bound for the Castelnuovo-Mumford regularity of subspace arrangements. Adv. Math. 172 (2002), no. 2, 151–157.
  Information about the free resolutions of intersections and products of ideals of linear forms.
- IT: Eisenbud, David; Hochster, Melvin: A Nullstellensatz with nilpotents and Zariski's main lemma on holomorphic functions.J. Algebra 58 (1979), no. 1, 157–161.
  Any ideal in an affine ring is the intersection of powers of the maximal ideals containing it.
- IT, AG: Eisenbud, David; Evans, E. Graham, Jr.: Every algebraic set in n-space is the intersection of n hypersurfaces. Invent. Math. 19 (1973), 107–112.
  The title says it all.
- IT, HA: Eisenbud, David: Some directions of recent progress in commutative algebra. Algebraic geometry (Proc. Sympos. Pure Math., Vol. 29, Humboldt State Univ., Arcata, Calif., 1974), pp. 111–128. Amer. Math. Soc., Providence, R.I., 1975.
  Sketches three interesting theorems in commutative algebra:
  (1) Projective modules over a polynomial ring are free (Quillen-Suslin);
  (2) Existence of big Cohen-Macaulay modules and applications (Hochster);
  (3) Hilbert-Burch theorem on ideals of codimension 2 and applications; for example, a regular local ring of dimension three modulo an element is factorial iff the element is not a determinant (Andreotti-Frankel).
- IT: Goto, Shiro: Integral closedness of complete-intersection ideals. J. Algebra 108 (1987), no. 1, 151–160.
  When can a parameter ideal be integrally closed?
- IT: Hochster, M.: Properties of Noetherian rings stable under general grade reduction. Arch. Math. (Basel) 24 (1973), 393–396.
  What can you expect when you factor out a generic element?
- IT: Hochster, Melvin: The Zariski-Lipman conjecture in the graded case. J. Algebra 47 (1977), no. 2, 411–424. (First part of paper)
  For a nice affine ring, the module of differentials is free iff the ring is regular; what about the dual of the module of differentials?

- IT: Johnston, Bernard; Katz, Daniel: Castelnuovo regularity and graded rings associated to an ideal. Proc. Amer. Math. Soc. 123 (1995), no. 3, 727–734.

  Relation between the Cohen Macaulay property for the Rees algebra and the associated graded ring of an ideal.
- IT, HA: Kunz, Ernst: Almost complete intersections are not Gorenstein rings. J. Algebra 28 (1974), 111–115.

  The title says it all.
- IT: Northcott, D. G.; Rees, D.: Reductions of ideals in local rings. Proc. Cambridge Philos. Soc. 50, (1954). 145–158.

  Classic paper: when is an ideal in the integral closure of a smaller ideal.
- IT : Reid, Les; Roberts, Leslie G.; Roitman, Moshe: On complete intersections and their Hilbert functions. Canad. Math. Bull. 34 (1991), no. 4, 525–535.

  Proves "Hard Lefschetz Theorem" for certain complete intersections.
- IT, HF: Stanley, Richard P.: On the Hilbert function of a graded Cohen-Macaulay domain. J. Pure Appl. Algebra 73 (1991), no. 3, 307–314.

  Macaulay characterized the Hilbert functions of all standard graded algebras; this paper says which ones can be Hilbert functions of nice integral domains.
- IT: Valabrega, Paolo; Valla, Giuseppe: Form rings and regular sequences. Nagoya Math. J. 72 (1978), 93–101.

  A criterion for a regular sequence in I to be regular in the associated graded ring of I.
- IT: Vasconcelos, Wolmer V.: Ideals generated by R-sequences. J. Algebra 6 1967 309–316.

  If I/I^2 is free, and I has finite projective dimension, then I is generated by a regular sequence.
- IT: Yao, Yongwei: Primary decomposition: compatibility, independence and linear growth. Proc. Amer. Math. Soc. 130 (2002), no. 6, 1629–1637

  A compatibility result for primary decompositions. The P-primary component from any decomposition can be used in any other decomposition.

Algebraic Geometry.

- AG Artin, Michael: On isolated rational singularities of surfaces. Amer. J. Math. 88 1966 129–136.

  Studies the simplest singularities—rational double points.
- AG Brieskorn, Egbert: Examples of singular normal complex spaces which are topological manifolds. Proc. Nat. Acad. Sci. U.S.A. 55 1966 1395–1397. (Reviewer: F. Hirzebruch))

  The manifolds homeomorphic but not diffeomorphic to a sphere, presented as as algebraic varieties.

- AG, HF, IT: to: Theorie des intersections et théorème de Riemann-Roch. Seminaire de Géometrie Algebrique du Bois-Marie 1966–1967, Dirigé par P. Berthelot, A. Grothendieck et L. Illusie. Avec la collaboration de D. Ferrand, J. P. Jouanolou, O. Jussila, S. Kleiman, M. Raynaud et J. P. Serre. Lecture Notes in Mathematics, Vol. 225. Springer-Verlag, Berlin-New York, 1971. Chapter by Kleiman on "Les Theoremes de Finitude pour le Foncteur de Picard".

  If you give the first two coefficients of the Hilbert function of a homogeneous domain there are only finitely many possibilities for the rest.
- AG, IT: Sturmfels, Bernd; Trung, Ngo Viet; Vogel, Wolfgang: Bounds on degrees of projective schemes. Math. Ann. 302 (1995), no. 3, 417–432.

  Bounds for the arithmetic degree of monomial ideals and geometric degree of homogeneous prime ideals. (A Bézout theorem in special cases that includes more primary components.)

Characteristic p.

- CP: Herzog, Jüergen: Ringe der Charakteristik p und Frobeniusfunktoren. Math. Z. 140 (1974), 67–78.

  Gives the converse to a result of Peskine and Szpiro that Frobenius is "exact" on modules of finite projective dimension. Recovers a classic theorem of Kunz listed just below this.
- CP: Hochster, Melvin; Huneke, Craig: Tight closure. Commutative algebra (Berkeley, CA, 1987), 305–324, Math. Sci. Res. Inst. Publ., 15, Springer, New York, 1989.

  Introduces tight closure theory and gives applications.
- CP, HA: Huneke, Craig; Lyubeznik, Gennady: Absolute integral closure in positive characteristic. Adv. Math. 210 (2007), no. 2, 498–504.

  The absolute integral closure in characteristic p is Cohen-Macaulay.
- CP, IT: Katzman, Mordechai: Finiteness of $\bigcup_e \operatorname{Ass} F^e(M)$ and its connections to tight closure. Illinois J. Math. 40 (1996), no. 2, 330–337.

  Constructs an example of an ideal I such that the union of the associated primes of its Frobenius powers is infinite.
- CP, GA, AG: Knop, F.: Die Cohen-Macaulay-Eigenschaft eines Invariantenrings. Available at http://math.rutgers.edu/ knop/papers/CM.html

  Gives a short proof of the Hochster-Roberts theorem that invariants of reductive groups are Cohen-Macaulay, using ideas from tight closure theory.
- CP: Kunz, Ernst: Characterizations of regular local rings for characteristic p. Amer. J. Math. 91 1969 772784.

  Proves that the Frobenius morphism is flat iff the ring is regular.
- CP: Monsky, P.: The Hilbert-Kunz function. Math. Ann. 263 (1983), no. 1, 43–49.

  Proves the existence of the Hilbert-Kunz multiplicity.

Group Actions.

- GA: Chevalley, Claude: Invariants of finite groups generated by reflections. Amer. J. Math. 77 (1955), 778–782.

  Proves that such rings of invariants are polynomial rings, generalizing the case of symmetric functions. See also survey by Flatto, below.
- GA: Derksen, Harm: Degree bounds for syzygies of invariants. Adv. Math. 185 (2004), no. 2, 207–214.

  The title says it all.
- GA: Flatto, Leopold: Invariants of finite reflection groups. Enseign. Math. (2) 24 (1978), no. 3-4, 237–292.

  A survey, to go along with the paper of Chevalley.
- GA: Mukai, S.: Counterexample to Hilbert's fourteenth problem for the 3-dimensional additive group. RIMS preprint 1343,

  Available at http://www.kurims.kyoto-u.ac.jp/preprint/file/RIMS1343.pdf.

  Improvements of Nagata's construction of additive representations with non finitely generated rings of invariants. Connections to Rees algebras, coxeter groups, elliptic curves. . . .
- GA, IT: Roberts, Paul: An infinitely generated symbolic blow-up in a power series ring and a new counterexample to Hilbert's fourteenth problem. J. Algebra 132 (1990), no. 2, 461–473.

  An example of non finite generation of rings of invariants, approached from a Rees algebra point of view.
- GA, RT: Roberts, Paul: Abelian extensions of regular local rings. Proc. Amer. Math. Soc. 78 (1980), no. 3, 307–310.

  Proves that the integral closure of a regular local ring in an abelian Galois extension field is Cohen-Macaulay.
- GA: Steinberg, Robert: Nagata's example. Algebraic groups and Lie groups, 375–384, Austral. Math. Soc. Lect. Ser., 9, Cambridge Univ. Press, Cambridge, 1997.

  Example of an additive group action whose invariants are not finitely generated. See also Mukai, below.

Gröbner Bases and Hilbert Functions.

- GB, HA: Bayer, David; Stillman, Michael: A criterion for detecting m-regularity. Invent. Math. 87 (1987), no. 1, 1–11.

  Shows that you can compute the regularity of an ideal as the regularity of a generic initial ideal in revlex.
- HF: Bigatti, Anna Maria: Upper bounds for the Betti numbers of a given Hilbert function. Comm. Algebra 21 (1993), no. 7, 23172334.

  The title says it all.

- GB, HA: Caviglia, Giulio; Sbarra, Enrico: Characteristic-free bounds for the Castelnuovo-Mumford regularity. Compos. Math. 141 (2005), no. 6, 1365–1373.

  Gives a nice characteristic-free proof of bounds on Castelnuovo-Mumford regularity.
- GB: Green, Mark: Restrictions of linear series to hyperplanes, and some results of Macaulay and Gotzmann. Algebraic curves and projective geometry (Trento, 1988), 76–86, Lecture Notes in Math., 1389, Springer, Berlin, 1989.

  The first two sections give nice proofs of Macaulay's theorem and other basic results.

Homological Algebra.

- HA Auslander, Maurice: Modules over unramified regular local rings. Illinois J. Math. 5 1961 631–647.

  Proves the rigidity of Tor over unramified regular local rings.

  Avramov, Luchezar L.: Infinite free resolutions. Six lectures on commutative algebra (Bellaterra, 1996), 1118, Progr. Math., 166, Birkh¡8a¿user, Basel, 1998.

  Survey of what is known on infinite free resolutions.
- HA: Avramov, Luchezar and Buchweitz, Ragnar, Lower bounds for Betti numbers. Compositio Math. 86 (1993), no. 2, 147–158.

  If R is a standard graded k-algebra, and M is a graded module of finite length and finite projective dimension with $\sum_i 1^i dim_k(M_i) = 0$, then the sum of the Betti numbers of M is at least 2^d.
- HA: Avramov, Luchezar L.; Lescot, Jack: Bass numbers and Golod rings. Math. Scand. 51 (1982), no. 2, 199–211 (1983).

  Growth of the injective resolution of the residue field.
- HA Bruns, Winfried: "Jede" endliche freie Auflösung ist freie Auflösung eines von drei Elementen erzeugten Ideals. J. Algebra 39 (1976), no. 2, 429–439. ("Every" finite free resolution is a free resolution of an ideal generated by three elements) Available in English translation by Soumya Deepta Sanyal, arXiv:1012.5551

  This paper needs basic element theory. See the paper "Generating Modules Efficiently…" in the section on module theory below.
- HA: Eisenbud, David: Homological algebra on a complete intersection, with an application to group representations. Trans. Amer. Math. Soc. 260 (1980), no. 1, 35–64.

  Sections 5 and 6 describe the Cohen-Macaulay modules over hypersurfaces in terms of matrix factorizations.
- HA: Eisenbud, David; Green, Mark L.: Ideals of minors in free resolutions. Duke Math. J. 75 (1994), no. 2, 339–352.

  Fitting ideals and properties of infinite free resolutions. Proves a conjecture of Huneke.

- HA: Gasharov, Vesselin N.; Peeva, Irena V.: Boundedness versus periodicity over commutative local rings. Trans. Amer. Math. Soc. 320 (1990), no. 2, 569–580.

  Shows that there are infinite resolutions with constant Betti numbers that are not periodic.

- HA: Heitmann, Raymond C.: A counterexample to the rigidity conjecture for rings. Bull. Amer. Math. Soc. (N.S.) 29 (1993), no. 1, 94–97.

  Auslander proved that the vanishing of $Tor_1^R(M,N)$ implies the vanishing of the $Tor_i^R(M,N)$ when R is regular and equicharacteristic (and somewhat more generally); this paper gives a counterexample in the case when we only assume that M has finite projective dimension.

- HA: Herzog, J.; Kühl, M.: On the Betti numbers of finite pure and linear resolutions. Comm. Algebra 12 (1984), no. 13-14, 1627–1646.

  Betti numbers of a pure free resolution are determined by the degrees.

- HA, CP: Hochster, Melvin: Topics in the homological theory of modules over commutative rings. Expository lectures from the CBMS Regional Conference held at the University of Nebraska, Lincoln, Neb., June 24–28, 1974. Conference Board of the Mathematical Sciences Regional Conference Series in Mathematics, No. 24. Published for the Conference Board of the Mathematical Sciences by the American Mathematical Society, Providence, R.I., 1975.

  In particular see the proof of the existence of Big Cohen-Macaulay modules in characteristic p. (See also the paper of Eisenbud in the IT section.)

- HA, IT: Kodiyalam, Vijay: Asymptotic behaviour of Castelnuovo-Mumford regularity. Proc. Amer. Math. Soc. 128 (2000), no. 2, 407–411.

  The regularity of the power of an ideal are bounded by a linear function of the exponent. Also extended to symmetric powers of modules.

- HA: Koh, Jee; Lee, Kisuk: Some restrictions on the maps in minimal resolutions. J. Algebra 202 (1998), no. 2, 671–689.

  This gives new restrictions on the entries of maps in finite free resolutions.

- HA: Tate, John: Homology of Noetherian rings and local rings. Illinois J. Math. 1 (1957), 14–27.

  First paper on the resolution of the residue field of a local ring. See also the survey of Avramov in the Barcelona volume.

- HA: Wang, Hsin-Ju: On the Fitting ideals in free resolutions. Michigan Math. J. 41 (1994), no. 3, 587–608.

  Fitting ideals and properties of infinite free resolutions. Relates them to Jacobian ideals. See also the paper of Koh and Lee.

Monomial Ideals.

- MI, HA: Bayer, Dave; Peeva, Irena; Sturmfels, Bernd: Monomial resolutions. Math. Res. Lett. 5 (1998), no. 1-2, 31–46

Gives the minimal free resolution for "generic" monomial ideals.

- MI, HA: Eagon, John A.; Reiner, Victor: Resolutions of Stanley-Reisner rings and Alexander duality. J. Pure Appl. Algebra 130 (1998), no. 3, 265–275.

  Proves that a square-free monomial ideal is Cohen-Macaulay iff its Alexander dual has linear resolution.
- MI: Herzog, Jürgen: A generalization of the Taylor complex construction. Comm. Algebra 35 (2007), no. 5, 1747–1756.

  Gives a resolution of a sum of monomial ideals, given resolutions for each one.
- MI: Lyubeznik, Gennady: A new explicit finite free resolution of ideals generated by monomials in an R-sequence. J. Pure Appl. Algebra 51 (1988), no. 1-2, 193–195.

  Steps toward making the Taylor resolution minimal.

Module Theory.

- MT: Bongartz, Klaus: A generalization of a theorem of M. Auslander. Bull. London Math. Soc. 21 (1989), no. 3, 255–256.

  Proves that two modules of finite length are isomorphic iff the lengths are the same after applying Hom into modules of finite length. Can be used together with the paper of Guralnick to prove the stronger result that two modules are isomorphic iff the lengths are the same after applying Hom into modules of finite length.
- MT: Brodmann, M.: Asymptotic stability of $\mathrm{Ass}(M/I^nM)$. Proc. Amer. Math. Soc. 74 (1979), no. 1, 16–18.

  The title says it all.
- MT: Eisenbud, David; de la Peña, J. A.: Chains of maps between indecomposable modules. J. Reine Angew. Math. 504 (1998), 29–35.

  If $A \to B \to C \to \cdots$ is a chain of non-isomorphisms between indecomposable modules of finite length whose composition is nonzero, then the sequence of lengths of the modules is quite special.
- MT, HA: Evans, E. Graham; Griffith, Phillip: The syzygy problem. Ann. of Math. (2) 114 (1981), no. 2, 323–333.

  Proves that the k-th syzygy of a module of finite projective dimension is either free or of rank at least k.
- MT: R. M. Guralnick: Lifting homomorphisms of modules, Illinois Jour. of Math. 29 (1985), no 1, 153–156.

  Shows one can lift homomorphisms of modules modulo high powers of the maximal ideals to homomorphisms of the modules themselves. Together with Bongartz's paper can be used to prove two modules of finite length are isomorphic iff the lengths are the same after applying Hom into modules of finite length.

- MT: Gruson, L.: Dimension homologique des modules plats sur an anneau commutatif noethérien. (French) Symposia Mathematica, Vol. XI (Convegno di Algebra Commutativa, INDAM, Rome, 1971), pp. 243–254. Academic Press, London, 1973.

  If M is a faithful A-module, then every A-module admits a finite filtration with factors that are homomorphic images of sums of copies of M.
- MT, HA: Huneke, Craig; Leuschke, Graham J.: Two theorems about maximal Cohen-Macaulay modules. Math. Ann. 324 (2002), no. 2, 391–404.

  Uses Miyata's result to give a short proof of Auslander's theorem that if a Noetherian local ring has only finitely many isomorphism classes of indecomposable Cohen-Macaulay modules, then the ring has isolated singularities. (In the graded case, a complete classification is known (see Eisenbud, David; Herzog, Jürgen: The classification of homogeneous Cohen-Macaulay rings of finite representation type. Math. Ann. 280 (1988), no. 2, 347352.)
- MT: Kaplansky, Irving: Projective modules. Ann. of Math (2) 68 1958 372–377

  Proves that every projective module over a local ring is free (the case when the module is finitely generated is an easy consequence of Nakayama's Lemma. The general case is clever and beautiful.)
- MT, HA: Matlis, Eben: Injective modules over Noetherian rings. Pacific J. Math. 8 1958 511–528. (Reviewer: G. Azumaya) 18.00

  A ring is Noetherian if and only if every direct sum of injective modules is injective, and in this case the structure of injective hulls mimics primary decomposition. This is the original paper giving the structure of injective modules over Noetherian rings.
- MT, HA: Miyata, Takehiko: Note on direct summands of modules. J. Math. Kyoto Univ. 7 1967 65–69.

  Proves that sequences that look split are split. For an application, see Huneke-Leuschke.
- MT: Swan, Richard G.: The number of generators of a module. Math. Z. 102 1967 318–322.

  Bounds the number of generators of a module in terms of the numbers of generators required for various localizations of the module. This is one of the special cases generalized in the theory of Basic Elements; see Eisenbud-Evans paper on that subject.

  Eisenbud, D. and Evans, E.G.: Generating modules efficiently: Theorems from algebraic K-theory. J. Algebra. 27 (1973), no.2, 278-305.

  Basic element theory is required for the paper of Bruns, "Jede freie Auflösung…" above. For another nice use of basic element theory, see

  Flenner, H.: Die Stze von Bertini fr lokale Ringe. Math. Ann. 229 (1977), no. 2, 97111.

  Strong form of the Bertini theorem that the general hyperplane section of a smooth variety is smooth. (Theorem 2.1: In a local ring a general element of the maximal ideal is not in the symbolic square of any prime, and more. . . .)

Ring Theory.

- RT: Claborn, Luther: Dedekind domains and rings of quotients. Pacific J. Math. 15 1965 59–64.

  Proves that any abelian group can occur as the ideal class group of a Dedekind domain. For the latest paper on this subject see Clark, Pete L.: Elliptic Dedekind domains revisited. Enseign. Math. (2) 55 (2009), no. 3-4, 213225.

- RT: Eisenbud, David: Subrings of Artinian and Noetherian rings. Math. Ann. 185 1970 247–249.

  Proves that a ring with a module-finite Noetherian ring extension is itself Noetherian.

- RT: Lech, Christer: A method for constructing bad Noetherian local rings. Algebra, algebraic topology and their interactions (Stockholm, 1983), 241–247, Lecture Notes in Math., 1183, Springer, Berlin,1986. (Reviewer: Luchezar L. Avramov) 13B35 (13C15 13H99)

  Shows that the ring of formal power series over any finite dimensional local algebra can be the completion of a local domain, and more. . . .

- RT: Matsumura, Hideyuki: On the dimension of formal fibres of a local ring. Algebraic geometry and commutative algebra, Vol. I, 261–266, Kinokuniya, Tokyo, 1988.

  Shows that if R is an affine domain with quotient field K, and $\hat{R}$ the completion of R at a maximal ideal, then $dim(K \otimes_R \hat{R}) = dim R - 1$.

- RT: Seidenberg, A.: Derivations and integral closure. Pacific J. Math. 16 1966 167–173.

  Every derivation of a domain R to itself in characteristic 0 also takes the integral closure of R to itself.