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Simulating symbolic distance effects in the transitive inference problem

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Abstract

The hippocampus is needed to store memories that are reconfigurable. Therefore, a hippocampal-like computational model should be able to solve transitive inference (TI) problems. By turning TI into a problem of sequence learning (stimuli-decisions-outcome), a sequence learning, hippocampal-like neural network solves the TI problem. In the transitive inference problem studied here, a network simulation begins by learning six pairwise relationships: A > B, B > C, C > D, D > E, E > F, and F > G where the underlying relationship is the linear string: A > B > C > D > E > F > G. The simulation is then tested with the novel pairs: B?D, C?E, D?F, B?E, C?F, B?F, and A?G. The symbolic distance effect, found in animal and human experiments, is reproduced by the network simulations. That is, the simulations give stronger decodings for B > F than for B > E or C > F and decodings for B > F and C > F are stronger than for B > D, C > E, or D > F.

Keywords: Recurrent networks; Sequence learning; Transitive inference; Symbolic distance effects; Hippocampus

1. Introduction

Transitive inference (TI) requires inferring a strictly linear ranking of symbols based on training that consists of only adjacent pairs. The symbolic distance effect in TI is the observation that novel pairings are solved better (i.e. correct relative ranking is achieved with higher accuracy and/or shorter reaction time) if the tested items are further apart in the implied series [10]. Thus, given the linear ranking string (which is

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made up of arbitrary stimuli, but which we will describe with sequential, alphabetical characters ABCDEFG), subjects typically solve the test pair B?F better than either the C?E or B?D pair after training on seven or more items (e.g. this result obtains in pigeons [2] and in humans [10]).

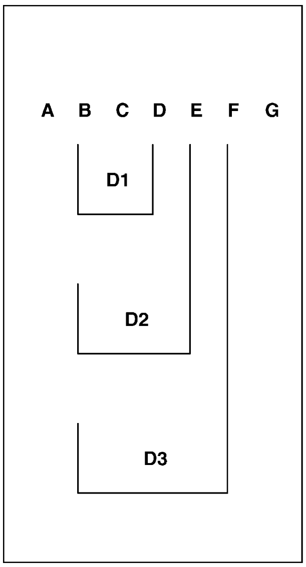
We have previously shown that a hippocampal-like neural network model can solve a transitive inference problem [8,13] consisting of five items and four relationships, A > B, B > C, C > D, D > E, and testing B?D. Here, in order to study the symbolic distance effect, we extend the study by training on seven items and the necessary six pairwise relationships as studied in psychological experiments.

2. The problem of transitive inference and symbolic distance effects

The TI problem is regarded as a logic problem in cognitive psychology (see Ref. [11]). TI is based on a pairwise ordering relationship such as "better than". TI is said to develop if subjects can infer that A is better than C from the relationships A is better than B and B is better than C.

In the transitive inference problem studied here, seven atomic stimuli (e.g. A, B, C, D, E, F, and G) are used. When A and B are together, A is the better answer (A > B). When B and C are together, B is the better answer (B > C) and so on. Thus, training

Table 1
Symbolic distance in the transitive inference problem. The symbolic distance between BD, CE, DF is one (1) symbol, so we define these pairs as the D1 group. Likewise, D2 group members are BE and CF. The D3 group has only one member, BF. Psychological experiments find that performance is better the greater the symbolic distance (e.g. [15])



D1	D2	D3
BD	BE	BF
CE	CF	
DF		

consists of A > B, B > C, C > D, D > E, E > F, and F > G. The critical tests for TI after learning consist of testing untrained pairings. Note that not all such test pairings are equally difficult to solve. Three pairings, B?D, C?E, D?F, are separated by one intervening symbol; two pairings are separated by two intervening symbols, B?E, C?F; and one pairing is separated by three intervening symbols (B?F). Test with end items are not counted because they can be solved without transitive inference. That is, these pairings test TI at three symbolic distances: responses for distance one (D1): B?D, C?E, D?F; the responses for distance two (D2): B?E, C?F; and for distance three (D3): B?F (see Table 1).

3. The model

The hippocampal model is essentially a model of region CA3 [5]. The input layer corresponds to a combination of the entorhinal cortex and dentate gyrus. To establish the sufficient nature of region CA3, decoding is performed by similarity comparisons rather than a CA1-subiculum-entorhinal decoding system. The CA3 model is a sparsely (10%) interconnected feedback network of 2048 neurons where all direct connections are excitatory and the network elements are McCulloch–Pitts neurons. There is an interneuron mediating feedforward inhibition, and one mediating feedback inhibition. Inhibition is of the divisive form, but the system is not purely competitive because of a slight delay in adjusting inhibition from time step to time step. Synaptic modification occurs during training. The process controlling synaptic modification is a local, self-adaptive postsynaptic rule that includes both potentiation and depression aspects [3,6]. The network computations are all local and are contained in three equations: spatial summation adjusted by inhibition; threshold to fire or not; and local Hebbian synaptic modification (See e.g. [4,5,7,12,14] for details).

4. Methods

Lesion experiments in rats suggest that TI is a hippocampally dependent function [1]. As part of our hypothesis [4,5] of the computation performed by the hippocampus, we have cast the TI problem as sequence prediction, whereby sequences in the form stimulus-decision-reinforcement are presented to the network using a staged learning paradigm (see [1]). Hence, we train the network on 12 sequences using a staged learning paradigm [1]. Each sequence is nine patterns long.

$$(AB)(AB)(AB)aaa +++ ,(AB)(AB)(AB)bbb ---,$$

$$(BC)(BC)(BC)bbb +++ ,(BC)(BC)(BC)ccc ---,$$

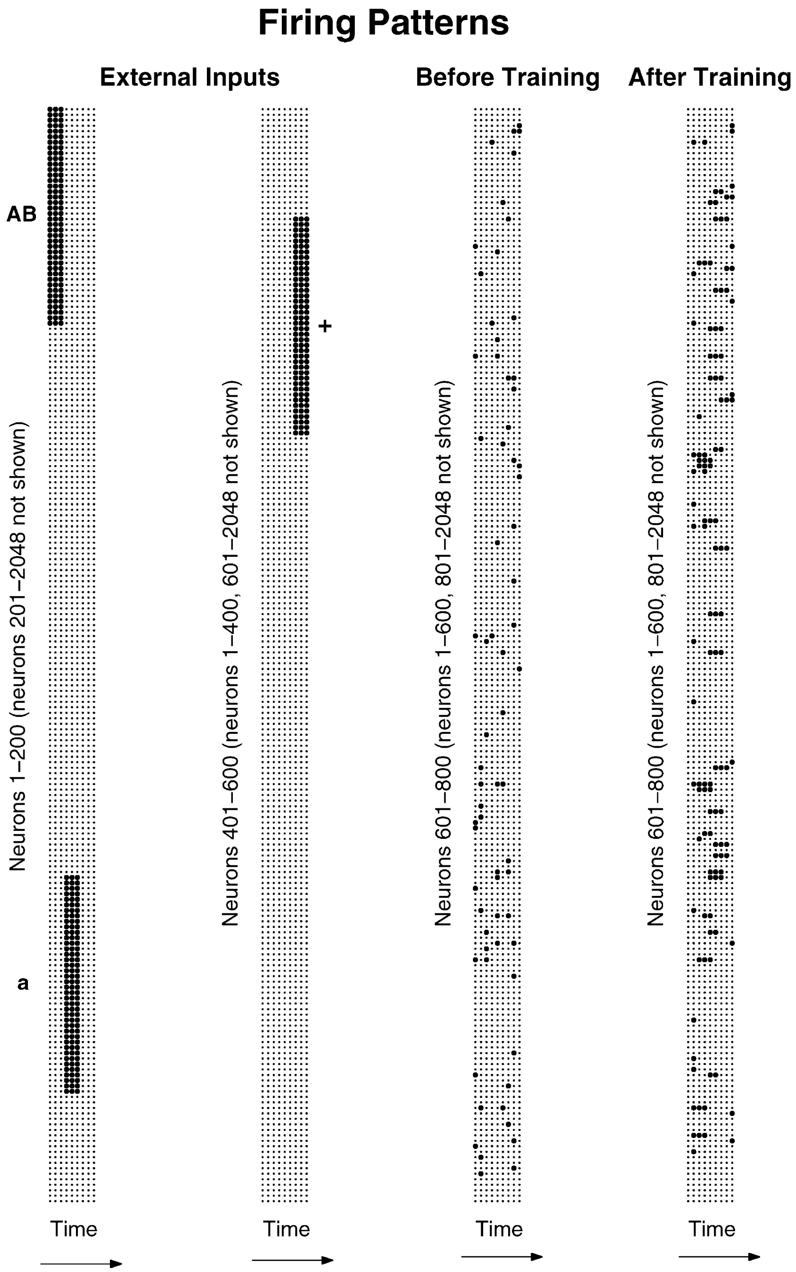
$$(CD)(CD)(CD)ccc +++ ,(CD)(CD)(CD)ddd ---,$$

$$(DE)(DE)(DE)ddd +++ ,(DE)(DE)(DE)eee ---,$$

$$(EF)(EF)(EF)eee +++ ,(EF)(EF)(EF)fff ---,$$

$$(FG)(FG)(FG)fff +++ \quad \text{and} \quad (FG)(FG)(FG)ggg ---,$$

where lower case letters correspond to decisions (e.g. 'aaa' means choose A) and +/− symbols correspond to the desirable and undesirable outcomes. During testing, we turn on part of the goal ' + ' to induce the model to follow the path in state space that leads to the right answer. This method is consistent with the concept



that most problem solving is performed with a desire for the correct answer [8,9](See Fig. 1). A test trial consists of a random input followed by activating a stimulus pair for three time steps all the while activating a fraction of the + input code, an output input pattern. Then the network is allowed to run, generating its own sequence of cell firings. We decode this sequence to discover the network's prediction. Decoding is done using a cosine comparison where we compare each state during testing to each state on the last training trial. For example, during testing, when BD and a fraction of the + input pattern are presented, we determine if the network produces sequence completion with a 'b' or a 'd' coding in the decision portion of the sequence.

5. Normalized difference in decoding strength

The "normalized difference in decoding strength" is defined as the cosine difference divided by the maximal cosine. For example, the normalized difference between the *b* and *d* codes is:

$$\frac{\cos(code_{test}, code_b) - \cos(code_{test}, code_d)}{\max(\cos(code_{test}, code_b),\cos(code_{test}, code_d))}.$$

Thus, the bigger the normalized difference, the stronger the decisions.

6. Results

Table 2 shows the network simulation results of the TI problem. As noted, the BD, CE, DF, BE, CF, BF tests rather than the AG test are critical here because A is always the right answer and G is always the wrong answer. From Table 2, one can see that B is the typically chosen answer when the BD pair is tested (which is not presented during learning). Likewise, B is the typically chosen answer when the BF pair is tested.

Importantly, the distance effects found in behavioral experiments [15] are reproduced by the simulations. For example, the network performance on B?F tests are better than on C?E tests; on B?F they are better than on C?F tests; on B?E tests, they

Fig. 1. An exemplary set of firing patterns for the transitive inference problem. Because the network is randomly connected, the spatial juxtaposition of any two neurons is irrelevant to network performance and is just used for explanatory convenience. Illustrated here is the input sequence of an AB trial (**A**, neurons 1–20; **B**, neurons 21–40; the decision input pattern **a** is represented by neurons 141–180. The + outcome input pattern given for the correct response is represented by activating neurons 421–460). The two vertical strips on the left-hand panels show a subset of the inputs (400 out of 2048 neurons) for the AB trial, (AB)(AB)(AB)aaa + + +. The two right-hand panels show a subset of neural firings (200 out of 2048 neurons) for the AB trial before and after training. Note the formation of long, local context neural firings after training which are critical for context-dependent learning. A large dot indicates a cell's firing, and a small dot indicates a nonfiring.

Table 2
Results from network simulations of the transitive inference problem. As in the rat experiments, the model performs significantly above chance (50%) on the BD, CE, DF, BE, CF, BF, and AG comparisons. Note that network performance on BF (D3) is better than on CE (D1) or CF (D2); BE tends to be better than CE, and CF tends to be better than DF. SEM is the standard error of the mean for 30 simulations. 2048 neurons are used for the simulations, and the activity level is about 7%

Testing type	Premise pairs/probe test	Percent correct	SEM
Testing	AB	100.00	0.00
on	BC	96.00	2.89
the trained pairs	CD	93.33	4.55
	DE	91.00	4.67
	EF	94.67	3.52
	FG	96.33	3.28
Testing	BD	95.33	3.39
on	CE	91.00	4.30
the untrained pairs	DF	83.00	6.43
	BE	96.67	3.28
	CF	90.33	4.56
	BF	99.67	0.33
	AG	100.00	0.00

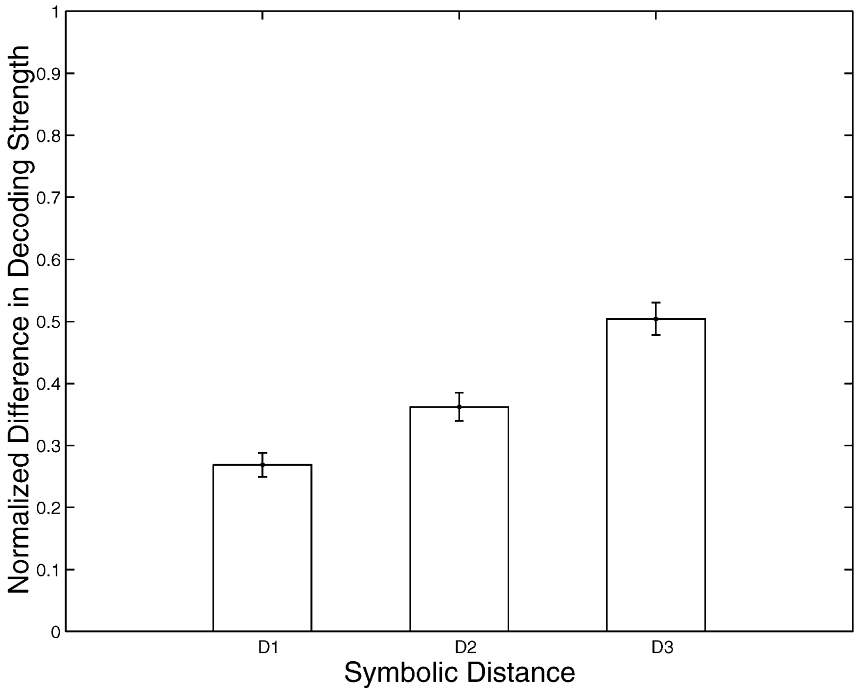


Fig. 2. Greater symbolic distance leads to stronger decisions. Error bars represent the standard error of the mean over 30 simulations. See Table 1 for definition of D1, D2, D3. See methods for normalized difference in decoding strength.

are better than on C?E tests; and on C?F they are better than on D?F tests, and this is also the trend of behavioral results.

Similar trends hold for the relative strength of the decodings. In Fig. 2, the normalized difference in decoding strength (see Methods for definition) is plotted for probe test pairs with different symbolic distance D1, D2, and D3 (the actual difference without normalization are about ten fold less, i.e. 0.035 ± 0.0035, 0.049 ± 0.0047, and 0.073 ± 0.0069 respectively). The differences are statistically significant whether normalized or not. That is, the simulations performed statistically better for D3 than D2, and D2 better than D1.

7. Discussion

A simple computational model of the hippocampus can reproduce some of the hallmark features of transitive inference. In this report, we have shown that a hippocampal-like network can not only solve the transitive inference problem but can also reproduce the symbolic distance effects observed in animal and human experiments. The promise of this model is to predict the neuronal firing patterns and connectivities that underlie the observed performance.

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