

Problem Set 9 Key: Multiple Regression

PLAD 710

Spring 2005

From Moore and McCabe

11.4 This suggests that, collectively, the quizzes were useful in explaining variation in the final exam score, but that no single quiz obtains a statistically significant relationship with the final exam. This is, perhaps, not too surprising, as quiz scores across the 10 quizzes are probably strongly correlated. Thus, holding the other 9 quizzes constant doesn't leave much variation in any given quiz available to explain variation in y . In other words, this is probably due to multicollinearity among the X (the different quizzes).

11.14 (a) The hypotheses: $H_0 : \beta_1 = \beta_2 = \dots = \beta_{13} = 0$ vs. $H_a : \text{not all slopes} = 0$.

The F -statistic of 27.97 has $df=(13, 5650)$; Since 13 and 5650 don't show up in the F -table, I'll use the closest smaller values (12 and 1000) – 27.97 > 2.77, the highest F value in the table for these df , so $p < .001$. We can reject the null that all slopes are jointly equal to zero.

(b) 14.1% of the variation in interest rates (y) is explained by these 13 independent variables (less than the 29.7% in 11.13 – but these values aren't strictly comparable, since the models are not estimated on the same data – bad M&M).

(c) The null for each of the j slopes is $H_0 : \beta_j = 0$, the alternatives vary by variable (and it looks like the estimated directions are probably in the alternative hypothesized directions). The df for these tests is $5664-13-1=5650$; this is so large, we can use the z row from the t table, so a value of t of 1.96 or greater will produce a p -value of .05 or less (for a two-sided alternative hypothesis), and thus a rejection of the null hypothesis.

(d) The variables for which the slope estimate is significantly different from zero are: loan size, length of loan, percent down payment, unsecured loan (bad credit or years at current address would make the cut if we used $p_{.10}$).

(e) Let's just take the significant coefficients. As loan size increases, the interest rate decreases. As the length of the loan increases, the interest rate decreases. As the percent down payment increases, the interest rate decreases. (In the presence of a bad credit report, the interest rate increases. And the longer the applicant has been at the same address, the lower the interest rate). All of these include the *ceteris paribus* caveat, of course.

Additional Problem-from Gujarati Ch 5

1. State whether the following statements are true, false, or uncertain. Briefly justify each answer.

- (a) The t test of significance requires that the sampling distributions of estimators $\hat{\beta}_0$ and $\hat{\beta}_1$, etc., follow the normal distribution.
True; the t test is based on variables with a normal distribution divided by RVs with a chi-square distribution.
- (b) Even if the disturbance term in the classic linear regression model is not normally distributed, the OLS estimators are still unbiased.
True; the normality assumption doesn't impact the property of unbiasedness (the Gauss-Markov assumptions – some of them – do)
- (c) The p value and the size of a test statistic (t , F) mean the same thing.
(d) I'd take either answer, appropriately justified. On the one hand, one may reach the same conclusion about the null from these two values; on the other hand, they aren't the same thing (for instance, the same t value can produce two different p -values, depending on whether your hypothesis test is one-sided or two-sided).
- (d) If a null hypothesis is not rejected, it is true.
False; failing to reject doesn't mean we accept the alternative, another alternative could also be true.
- (e) The higher the value of σ^2 , the larger is the variance of $\hat{\beta}_1$.
True; as σ^2 increases, so does $var(\hat{\beta}_1)$ – we can see this in the equation for the variance of slope coefficient in the bivariate case (equations 7 and 8 from April 14).
- (f) The conditional and unconditional means of a random variable are the same things.
False; the conditional mean is a function of X (or multiple X s); the (unconditional) mean is not.

Playing with Stata

- Using the dataset nes04lab2.dta, estimate the following simple model of candidate evaluation separately for each candidate (Kerry and Bush).

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \beta_5 X_{5i} + \beta_6 X_{6i} + \beta_7 X_{7i} + u_i \quad (1)$$

Where

- $Y_i = ftbush/ftkerry$, feeling thermometer for Gore or Bush (0=very cold/unfavorable, 50=neutral, 100=very warm/favorable)
- $X_{1i} = repub$, strength of Republican identification (0=none, 1=leaning Republican, 2=weakly Republican, 3=strongly Republican)
- $X_{2i} = democ$, strength of Democratic identification (0=none, 1=leaning Democratic, 2=weakly Democratic, 3=strongly Democratic)
- $X_{3i} = retecon$, retrospective national economic perceptions, or how do you think the economy's doing compared to last year (1=much better, 2=somewhat better, 3=same, 4=worse, 5=much worse)
- $X_{4i} = retunemp$, retrospective unemployment perceptions, or how is the level of unemployment compared to last year (1=much better, 2=somewhat better, 3=same, 4=worse, 5=much worse)

- $X_{5i} = \text{retinfl}$, retrospective inflation perceptions, or how is the level of inflation compared to last year (1=much better, 2=somewhat better, 3=same, 4=worse, 5=much worse)
- $X_{6i} = \text{age}$, respondent's age in years
- $X_{7i} = \text{income}$, respondent's annual household income, in dollars.

Candidate Evaluations

	Kerry	Bush
Strength of Republican Identification	-7.494* (0.762)	10.114* (0.915)
Strength of Democratic Identification	6.561* (0.746)	-8.201* (0.897)
Perceptions of Economy	2.639* (0.821)	-6.213* (0.987)
Perceptions of Unemployment	2.449* (0.801)	-1.538 (0.962)
Perceptions of Inflation	0.304 (0.881)	-1.739 (1.057)
Age	-0.013 (0.040)	0.173* (0.048)
Income	-0.00001 (0.00001)	-0.00007* (0.00002)
Constant	36.208 (4.079)	81.824 (4.910)
R^2	0.476	0.538
N	889	895

Note: * denotes $p < .01$.

2. Interpret the effects of the independent variables on evaluations for each candidate (verbally summarize the relationship; be specific).

For Kerry, as strength of Republican identification increases, evaluations decline (strong Republicans, on average, evaluate Kerry about 22 points less favorably than do non-Republicans). Similarly, as Democratic identification increases, evaluations of Kerry increase (strong Democrats, on average, evaluate Kerry about 19 points more favorably than do non-Democrats). As one's perceptions of the national economy and unemployment get worse (much better is 1, much worse is 5), evaluations of Kerry increase. Add a big *ceteris paribus* to all that. (I could interpret the remaining IVs, but as they aren't significant, I'm going to pass).

For Bush, as strength of Republican identification increases, evaluations increase (strong Republicans, on average, evaluate Bush about 30 points more favorably than do non-Republicans). Similarly, as one moves toward stronger Democratic identification, evaluations of Bush fall, on average (so that strong Democrats are around 24 points less favorable toward Bush). The worse one thinks the economy is, the less favorably one feels toward Bush. As age increases, so do evaluations of Bush, on average (but this effect is relatively small; for every 10 additional years of age, folks evaluate Bush just under 2 points more favorably). As income increases,

evaluations of Bush, on average, decrease (this effect, too, is quite small, as it takes a \$100,000 increase in income to move the average evaluation down about 6 or 7 points). The worse one perceived unemployment and inflation to be, the less positively one evaluates Bush – but as these coefficients are arguably insignificant, I wouldn't make a scene about it.

3. For each model, are the coefficients individually significant? If so, at what level of significance?

For Kerry, strength of Republican and of Democratic identification are significantly related to evaluations, as are retrospective perceptions of the economy and unemployment. Income, age, and retrospective inflationary perceptions are not discernible from zero.

For Bush, strength of Republican and of Democratic identification are significantly related to evaluations, as are retrospective perceptions of the economy. Retrospective perceptions of unemployment and inflation are more debatable – probably I would fail to reject the null in these cases. The relation between Bush's evaluations and age and income, though, are statistically significant.

4. For each model, are the coefficients in the model jointly significant? How do you know?

Yes, the overall F -tests for both models are quite high, resulting in very low p -values in both cases. The null that all coefficients are jointly equal to zero can be strongly rejected for both models.

5. Overall, how well does this model fit the data for Kerry? For Bush?

For Kerry: The adjusted R^2 is 0.4762, a reasonable fit.

For Bush: The adjusted R^2 is 0.5379, a reasonable fit (which appears a little stronger than for Kerry, but these R^2 are not strictly comparable because the models use different dependent variables, and slightly different cases, so I wouldn't make too, too much of this).

6. For each model, are the effects of retrospective national economic perceptions and retrospective unemployment perceptions equal? (Use the test command, and report the appropriate statistics in your answer) How do you interpret your answer (e.g., what does this mean substantively)?

For Kerry: I cannot reject the null that the two coefficients are equal (unsurprising, as they look very similar; the F is 0.02, with a p -value of 0.8927).

For Bush: I can reject the null that the two coefficients are equal – perceptions of the overall economy appear to matter noticeably more than perceptions of unemployment (F is 7.63, with a p -value of 0.0058).

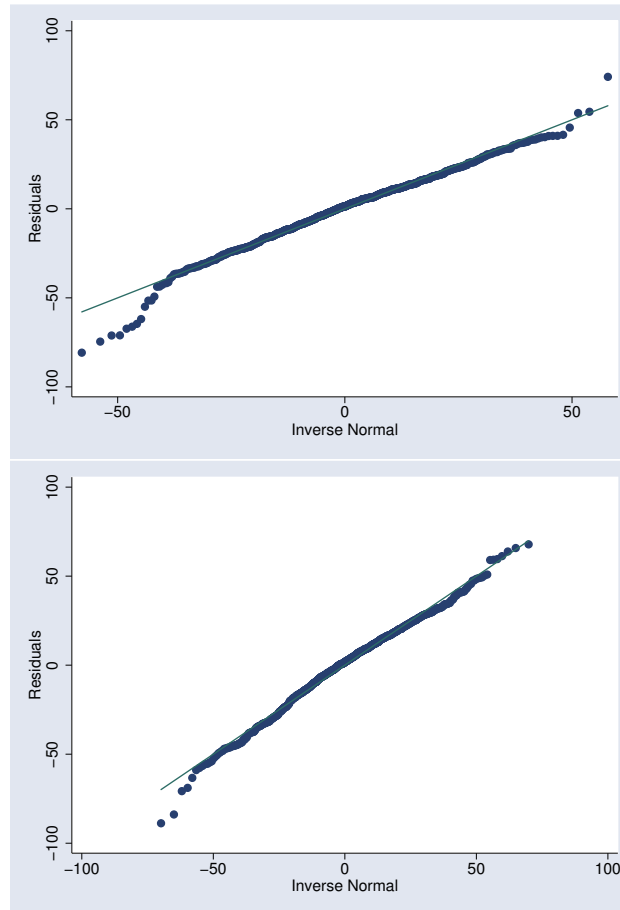
7. For each model, is the effect of being a Republican equal and *opposite* to the effect of being a Democrat? (Use the test command, and report the appropriate statistics in your answer) How do you interpret your answer?

For Kerry: I cannot reject the null that the two coefficients are equal in magnitude and opposite in sign (F is 0.49 with a p -value of 0.4827).

For Bush: I cannot reject the null that the two coefficients are equal in magnitude and opposite in sign (F is 1.44 with a p -value of 0.2309).

8. For each model, obtain the residuals. Produce a graph of the residuals to examine the distribution. What do you conclude?

They both look pretty good – there’s a little more left-tail skew in the Kerry residuals, but no stark deviations from normality in either set.



9. For each model, graph the residuals against some X of your choosing. What do you conclude?

Shown, Kerry residuals against income; Bush residuals against age. Frankly, they don't look too bad (the most patterned scatterplots appear to be Kerry residuals against repub, and Bush residuals against repub and democ – each of these indicate, perhaps, some correlation between these X s and the relevant disturbances.)

