# Switching Equilibria*

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May 4, 2006

**Abstract**

After an election, when party positions and strengths are known, there may be a centrally located large party at the core position. Whether this is so can be determined by examining the McKelvey-Schofield symmetry conditions at the point. If the conditions are satisfied, then this party will typically be able to form a minority government and control policy. In the absence of a core party, theory suggests that the outcome can be assumed to generated by a lottery associate with coalition risk.

Models of elections typically indicate that all parties, in equilibrium, will adopt positions at the electoral center. This situation, however, is seldom observed. This paperer outlines how an integrated theory of party strategy may be constructed based on an existence theorem for existence of Local Nash Equilibium (LNE). The concept allows for the balancing of office and policy motivation. The LNE is said to be structurally stable (ss) if it qualitatively insensitive to party agents or subgroups changing party allegiance. This notion can then be used to provide an account of the differing motivations for party switching. The model is used to interpret recent examples of party switching in the Israel Knesset.

*This paper is based on research supported by NSF Grant SES 024173. Earlier related research was conducted in collaboration with Andrew Martin, Gary Miller, David Nixon, Robert Parks, Kevin Quinn, Itai Sened and Andrew Whitford.

# 1 Structurally Stable Local Nash Equilibria as a Model for Party Switching.

A standard model of political competition is one where party leaders adopt positions to maximize vote share in the context of an electorate whose preferences exhibit a stochastic element. For example,Hinich (1977) argued that vote maximizing candidates would adopt a position at the mean of the voter distribution. His argument for two-party competition has been extended by Enelow and Hinich (1984,1989), Coughlin (1992) and most recently by McKelvey and Patty (2005) and Banks and Duggan (2005). Lin, Enelow and Dorussen(1999) have also obtained a "mean voter theorem", for the general case of many candidates.

Applying a stochastic model of voting is the standard technique for estimating voter response in empirical analyses ( Alvarez and Nagler, 1998; Alvarez, Nagler and Bowler, 2000). In an early application it was noted by Poole and Rosenthal(1984) that there was no evidence of convergence to the electoral mean in US presidential elections. Recently, empirical analyses of party competition in the US, Britain, Germany, the Netherlands and Israel have constructed "stochastic" spatial electoral models and found divergence of party positions away from the electoral mean .[1]

These empirical models have all entailed the addition of heterogeneous intercept terms for each party. One interpretation of these intercept or constant terms is that they are valences or party biases. "Valence " refers to voters' judgements about positively or negatively evaluated aspects of candidates, or party leaders, which cannot be ascribed to the policy choice of the party or candidate (Stokes, 1992). One may conceive of the valence that a voter ascribes to a candidate as a judgement of the candidate's quality or competence. This idea of valence has been utilized in a number of recent formal models of voting (Ansolabehere and Snyder, 2000; Groseclose, 2001; Aragones and Palfrey, 2002). .

The next section of this paper presents such a characterization, in terms of the Hessian of the vote share function of the party leader or candidate who has the lowest valence. For the case when the stochastic errors have the Type I extreme value (or log Weibull) distribution,$\Psi$. Theorem 1 shows that there exists a "convergence coefficient "which is a function of all the parameters of the model. A sufficient condition for the existence of a "local

[1] See Schofield, Martin,Quinn,and Whitford; 1998; Schofield, Sened and Nixon, 1998; Quinn, Martin and Whitford, 1999.Schofield and Sened, 2002, 2005a,b,2006; Miller and Schofield, 2003; Schofield, Miller and Martin,2003; Schofield, 2005a,b.

Pure Strategy Nash equilibrium" at the electoral mean is that this coefficient is bounded above by 1. When the policy space is of dimension $w$, then the necessary condition for existence of a Pure Strategy Nash Equilibrium at the electoral mean, and thus for the validity of the "mean voter theorem", is that the coefficient is bounded above by $w$.In the two dimensional case, the eigenvalues of the Hessian can be computed. It is shown that the convergence coefficient is (i) an increasing function of the maximum valence difference (ii) an increasing function of the number of parties or candidates and (iii) an increasing function of the electoral variance of the voter preferred points . Similar analyses can be carried out under the more generaal assumption that the stochastic distribution is multivariate normal, and in this case the coefficient is a decreasing function of the stochastic variance.

When the necessary "convergence condition" fails, then the origin will be a saddlepoint or minimum of the vote share function for the lowest valence party. By changing position in the major electoral axis (or eigenspace of the vote function) this party will increase vote share. It follows that in equilibrium, all parties will adopt positions on this principal axis, with the lowest valence parties the furthest from the origin. No party will adopt a position at the electoral mean. The empirical section of this paper shows that the convergence condition fails for an electoral model for the election of 1996 in Israel, but is astisfied for the model the Netherlands in 1979. . Simulation of the empirical model for Israel found that the vote maximizing positions of the parties were indeed not at the electoral mean . Although there was a close correspondence between the estimated actual positions of the parties and the equilibrium positions obtained by simulation, these positions were not identical.

These stochastic models all assume that the party leaders are motivated simply to maximize vote shares in order to gain office. Moreover, because the model focuses on expected vote share, it ignores the possibility of uncertainty in electoral reponse.One way to introduce uncertainty, at least in two-party models is to focus instead on the "probability of victory". Implicitly, such a model acknowledges the the vote share functions are stochastic variables. To extend such a model to the multiparty case, where there are three or more parties, requires a modification of the notion of "probability of winning". An obvious extension is to model electoral uncertainty in terms of the probabilities associated with different collections of decisive coalitions.The natural way to model party choice is then to allow party policy decisions to be made by party principals who have policy preferences. Section 3 models such policy-motivated choice using the idea of coalition risk and argues that the deviation of party position from vote maximizing equilibria in the Netherlands and Israel can be accounted for by the notion of coalition risk.

The primary purpose of the article is to present an equilibrium concept of party positioning in a formal model of elections that is potentially capable of explaining the rich diversity that can occur in multiparty polities. Because the proposed models are very precise about the consequences of changes in exogenous valence it is possible to draw out a number of conclusions about the political effect of party switching, or pre election coalition agreements.

The general conclusions that can be drawn from the formal models are:

(i) Under proportional rule, and with exogeneous valence alone,there will be a centripetal tendency that will make itself felt only for high valence parties. However, such parties will not converge *to* the electoral origin. As in Israel, if there are two opposed high valence parties, then they will locate on opposite sides of the mean, but on the principal electoral axis. For low valence parties, there will be a strong centrifugal tendency. Under vote maximization,such parties will locate along the "principal electoral axis". If they take up positions off this axis, then it will typically be because of the effect of coalition risk. This concept refers to the possibility that the position adopted by a low valence party will allow it to choose one or other of the high valence parties so as to constitute a governing coalition.This deviation is most noticeable in Israel, where Shas appears to locate itself in a position favorable to its ability to be pivotal in the formation of Likud based coalitions.

(ii) The paper uses the equilibrium notion of "local Nash equilibrium" (LNE). This concept allows for the balancing of the two different maximands-vote share and the ability to influence gvernment coalition. Although the local Nash equilibrium concept assumes that parties are well defined entities, the specification of the utility function of political agents allows for the determination of the equilibrium payoffs to such agents, given the pre-existing party membership.

(iv) The LNE is said to be *structurally stable* (*ss*) if it insensitive to permissible changes of party membership ( that is by individuals or subgroups switching party membership). Using this notion,It is then theoretically possible, to evaluate the LNE payoffs at various party associations to determine if the LNE is *ss*.A LNE that is not *ss* can then be destroyed by a switch of party allegiance. Some suggestions about the determinants of structural stability and instability are presented in a short conclusion. .. , .

## 2 Local Nash equilibrium with electoral certainty and office-motivated parties.

The purpose of this section is to construct a model of positioning of parties in electoral competition so as to account for the generally observed phenomenon of non-convergence .The model adopted is an extension of the multiparty stochastic model of Lin,Enelow and Dorussen(1999) ,constructed by inducing asymmetries in terms of valence. The basis for this extension is the extensive empirical evidence that valence is a significant component of the judgements made by voters of party leaders. There are a number of possible choices for the appropriate game form for multiparty competition . The simplest one, which is used here, is that the utility function for agent $j$ is proportional to the vote share ,$V_j$, of the agent. With this assumption, we can examine the conditions on the parameters of the stochastic model which are necessary for the existence of a pure strategy Nash equilibrium (PNE) for this particular game form. Because the vote share functions are differentiable, we use calculus techniques to estimate optimal positions. As usual with this form of analysis, wwe can obtain sufficient conditions for the existence of local optima, which we term local pure strategy Nash equilibria (LNE). Clearly, any PNE will be a LNE, but not conversely. Additional conditions of concavity or quasi-concavity are sufficient to guarantee existence of PNE. However, in the models we consider, it is evident that these sufficient conditions will fail, leading to the inference that PNE are typically non-existent. Existence of mixed strategy Nash equilibria is an open question in such games. It is of course true that the true utility functions of party leaders are unknown. However, comparison of LNE, obtained by simulation of empirical models, with the estimated positions of parties in the various polities that have been studied, can provide insight into the true nature of the game form of political competition.

The key idea underlying the formal model is that party leaders attempt to estimate the electoral effects of party declarations, or manifestos, and choose their own positions as best responses to other party declarations, in order to maximize their own vote share. The stochastic model essentially assumes that party leaders cannot predict vote response precisely. In the model with "exogenous" valence, the stochastic element is associated with the weight given by each voter, $i$, to the average perceived quality or valence of the party leader.

**Definition 1.** ***The Stochastic Vote Model.***

The data of the spatial model is a distribution, $\{x_i \in X\}_{i \in N}$ , of voter ideal points for the members of the electorate, $N$, of size $n$. As usual we assume that $X$ is a compact convex subset of Euclidean space, $\mathbb{R}^w$, with $w$ finite.Each of the parties,or agents, in the set $P = \{1, \ldots, j, \ldots, p\}$ chooses a policy, $z_j \in X$, to declare. Let $\mathbf{z} = (z_1, \ldots, z_p) \in X^p$ be a typical vector of agent policy positions. Given $\mathbf{z}$, each voter, $i$, is described by a vector $\mathbf{u}_i(x_i, \mathbf{z}) = (u_{i1}(x_i, z_1), \ldots, u_{ip}(x_i, z_p))$, where

$$u_{ij}(x_i, z_j) = \lambda_j + -\beta||x_i - z_j||^2 + \epsilon_j = u^*_{ij}(x_i, z_j) + \epsilon_j \tag{1}$$

Here $u^*_{ij}(x_i, z_j)$ is the observable component of utility. The term , $\lambda_j$ is the "exogenous" valence of agent $j$, $\beta$ is a positive constant and $||\cdot||$ is the usual Euclidean norm on $X$. The terms $\{\epsilon_j\}$ are the stochastic errors, whose cumulative distribution will be denoted by $\Psi$. In empirical analyses and in this paper it is assumed that $\Psi$ is the "extreme value Type I distribution" (sometimes called log Weibull). It is natural to suppose that the valence of party $j$, as perceived by voter $i$ is the stochastic variate $\lambda_{ij} = \lambda_j + \epsilon_j$, where $\lambda_j$ is the expectation $\mathcal{E}xp(\lambda_{ij})$ of $\lambda_{ij}$.We assume that the valence vector

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, .., \lambda_p) \text{ satisfies } \lambda_p \geq \lambda_{p-1} \geq \cdots \geq \lambda_2 \geq \lambda_1.$$

Because of the stochastic assumption, voter behavior is modeled by a probability vector. The probability that a voter $i$ chooses party $j$ is

$$\begin{aligned} \rho_{ij}(\mathbf{z}) &= \Pr[[u_{ij}(x_i, z_j) > u_{il}(x_i, z_l)], \text{ for all } l \neq j]. & (2) \\ &= \Pr[\epsilon_l - \epsilon_j < u^*_{ij}(x_i, z_j) - u^*_{il}(x_i, z_j), \text{ for all } l \neq j] & (3) \end{aligned}$$

Here Pr stands for the probability operator generated by the distribution assumption on $\epsilon$. The *expected vote share* of agent $j$ is

$$V_j(\mathbf{z}) = (1/n) \sum_{i \in N} \rho_{ij}(\mathbf{z}) \tag{4}$$

We shall use the notation $V : X^p \to \mathbb{R}^p$ and call $V$ the *party profile function.* In the vote model it is assumed that each agent $j$ chooses $z_j$ to maximize $V_j$, conditional on $\mathbf{z}_{-j} = (z_1, ..z_{j-1}, z_{j+1}.., z_p)$.

Because of the differentiability of the cumulative distribution function, the individual probability functions $\{\rho_{ij}\}$ are $C^2$-differentiable in the strategies $\{z_j\}$. Thus, the vote share functions will also be $C^2$-differentiable. Let $x^* = (1/n)\Sigma_i x_i$. Then the mean voter theorem for the stochastic model, asserts that the "joint mean vector" $\mathbf{z}_0 = (x^*, \ldots, x^*)$ is a "pure strategy Nash equilibrium". Lin, Enelow and Dorussen (1999) used $C^2$-differentiability of the expected vote share functions, in the situation with zero valence, to show that the validity of the theorem depended on the concavity of the vote share functions. They asserted that a sufficient condition for this was that $\sigma^2$ was "sufficiently large". Because concavity cannot in general be assured, we shall utilize a weaker equilibrium concept, that of "Local Strict Nash Equilibrium"(LSNE). A strategy vector $\mathbf{z}^*$ is a LSNE if , for each $j$, $z_j^*$ is a critical point of the vote function $V_j(z_1^*, ..z_{j-1}^*, z_j., z_{j+1}^*, ..z_p^*)$ and the eigenvalues of the Hessian of this function (with respect to $z_j$),are negative. Definition 2.1 gives the various definitions of the equilibrium concepts used throughout this book.

**Definition 2.*Equilibrium Concepts.***

(i) A strategy vector $\mathbf{z}^*=(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) \in X^p$ is a *local strict Nash equilibrium*(LSNE) for the profile function $V : X^p \to \mathbb{R}^p$ iff, for each agent $j \in P$,there exists a neighborhood $X_j$ of $z_j$ in $X$ such that

$$V_j(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) > V_j(z_1^*, ..., z_j..z_p^*) \text{ for all } z_j \in X_j - \{z_j^*\}$$

(ii) A strategy vector $\mathbf{z}^*=(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*)$ is a *local weak Nash equilibrium* (LNE) iff, for each agent $j$,there exists a neighborhood $X_j$ of $z_j$ in $X$ such that

$$V_j(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) \geq V_j(z_1^*, ..., z_j..z_p^*) \text{ for all } z_j \in X_j$$

(iii) A strategy vector $\mathbf{z}^*=(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*)$ is a *strict, respectively, weak* , *pure strategy Nash equilibrium* (PSNE, respectively, PNE) iff $X_j$ can be replaced by $X$ in (i),(ii) respectively .

(iv) The strategy $z_j^*$ is termed a "local strict best response", a "local weak best response", a "global weak best response", a "global strict best response",respectively to $\mathbf{z}_{-j}^*=(z_1^*, ...z_{j-1}^*, z_{j+1}^*..z_p^*)$.

Obviously if $\mathbf{z}^*$ is an LSNE or a PNE it must be an LNE, while if it is a PSNE then it must be an LSNE. We use the notion of LSNE to avoid problems with the degenerate situation when there is a zero eigenvalue to the Hessian. The weaker requirement of LNE allows us to obtain a necessary condition for $\mathbf{z}_0^* = (x^*, \ldots, x^*)$ to be a LNE and thus a PNE, without having to invoke concavity. The theorem below also gives a sufficient condition for the joint mean vector $\mathbf{z}_0^*$ to be an LSNE. A corollary of the theorem shows,in situations where the valences differ, that the necessary condition is likely to fail. In dimension $w$, the theorem can be used to show that, for $\mathbf{z}_0^*$ to be an LSNE, the necessary condition is that a "convergence coefficient ", defined in terms of the parameters of the model, must be strictly bounded above by

$w$. Similarly, for $\mathbf{z}_0^*$ to be a LNE, then the convergence coefficient must be weakly bounded above by $w$. When this condition fails, then the joint mean vector $\mathbf{z}_0^*$ cannot be a LNE and therefore cannot be a PNE. Of course, even if the sufficient condition is satisfied, and $\mathbf{z}_0^* = (x^*, \ldots, x^*)$ is an LSNE, it need not be a PNE.

To state the theorem ,we first transform coordinates so that in the new coordinates, $x^* = 0$. We shall refer to $\mathbf{z}_0 = (0,...0)$ as the *joint origin* in this new coordinate system. Whether the joint origin is an equilibrium depends on the distribution of voter ideal points. These are encoded in the voter covariation matrix. We first define this, and then use it to characterize the vote share Hessians.

**Definition 3: *The electoral covariance matrix,*** $\frac{1}{n}\nabla$. To characterize the variation in voter preferences, we represent in a simple form the covariation matrix (or data matrix), $\nabla$, given by the distribution of voter ideal points. Let $X$ have dimension $w$ and be endowed with a system of coordinate axes $(1, \ldots, r, s, \ldots, w)$. For each coordinate axis let $\xi_r = (x_{1r}, x_{2r}, \ldots, x_{nr})$ be the vector of the $r^{th}$ coordinates of the set of $n$ voter ideal points. We use $(\xi_r, \xi_s)$ to denote scalar product.

The symmetric $w \times w$ voter covariation matrix $\nabla$ is then defined to be

$$\nabla = ((\xi_r, \xi_s)) .$$

where $(\xi_r, \xi_s)$ is the scalar product of $\xi_r$ and $\xi_s$.The covariance matrix is defined to be $\frac{1}{n}\nabla$.

We write $v_s^2 = \frac{1}{n}(\xi_s, \xi_s)$ for the electoral variance on the $s^{th}$ axis and

$$v^2 = \sum_{r=1}^{w} v_r^2 = \frac{1}{n}\sum_{r=1}^{w}(\xi_r, \xi_r) = trace(\frac{1}{n}\nabla)$$

for the total electoral variance. The electoral covariance between the $r^{th}$ and $s^{th}$ axes is $(v_r, v_s) = \frac{1}{n}(\xi_r, \xi_s)$.

**Definition 4: *The Extreme Value Distribution,*** $\Psi$.

(i)The cumulative distribution has the closed form

$$\Psi(h) = \exp[-\exp[-h]],$$

with probability density function

$$\varphi(h) = \exp[-h] \exp[-\exp[-h] ,$$

and variance $\frac{1}{6}\pi^2$.

(ii) With this distribution it follows from Definition 4 , for each voter $i$, and party ,$j$,that

$$\rho_{ij}(\mathbf{z}) = \frac{\exp[u^*_{ij}(x_i, z_j)]}{\sum_{k=1}^{p} \exp u^*_{ik}(x_i, z_k)}. \tag{5}$$

Note that (ii) implies that the model satisfies the independence of irrelevant alternative property (IIA): for each individual $i$, and each pair,$j,k$, the ratio

$$\frac{\rho_{ij}(\mathbf{z})}{\rho_{ik}(\mathbf{z})}$$

is independent of a third party $l$( See Train, 2003,p.79)

While this distribution assumption facilites estimation , the IIA property may be violated. Below we consider the case of covariant errors, thus allowing for violation of IIA.

The formal model just presented, and based on $\Psi$ is denoted $M(\boldsymbol{\lambda},\beta;\Psi,\nabla)$, though we shall usually suppress the reference to $\nabla$.

**Definition 5. The Convergence Coefficient of the model** $M(\boldsymbol{\lambda},\beta;\Psi)$.

(i) At the vector $\mathbf{z}_0$ =(0,..0) the probability $\rho_{ik}($ $\mathbf{z}_0)$ that $i$ votes for party,$j$is

$$\rho_j = \left[1 + \sum_{k \neq j} \exp\left[\lambda_k - \lambda_j\right]\right]^{-1} \tag{6}$$

(ii) The coefficient $A_j$ for party $j$ is

$$A_j = \beta(1 - 2\rho_j)$$

(iii) The Hessian for party $j$ at $\mathbf{z}_0$ is

$$C_j = \left[2[A_j](\frac{1}{n}\nabla) - I\right]$$

where $I$ is the $w$ by $w$ identity matrix.

(iv) The *convergence coefficient of the model* $M(\boldsymbol{\lambda},\beta;\Psi)$ is

$$c(\boldsymbol{\lambda},\beta;\Psi) = 2\beta[1 - 2\rho_1]v^2 = 2A_1v^2 \ . \tag{7}$$

The definition of $\rho_j$ follows directly from the definition of the extreme value distribution. Obviously if all valences are identical then $\rho_1 = \frac{1}{p}$,as expected. The effect of increasing $\lambda_j$, for $j \neq 1$, is clearly to decrease $\rho_1$, and therefore to increase $A_1$,and thus $c(\boldsymbol{\lambda},\beta;\Psi)$.

**Theorem 1.** The condition for the joint origin to be a LSNE in the model $M(\boldsymbol{\lambda},\beta;\Psi)$ is that the Hessian

$$C_1 = \left[ 2[A_1](\frac{1}{n}\nabla) - I \right] \tag{8}$$

of the party 1, with lowest valence,has negative eigenvalues.□

*Comment on the Theorem.* The proof of the Theorem is given in Schofield (2006). The proof depends on considering the first and second order conditions at $\mathbf{z}_0$ for each vote share function. The first order condition is obtained by setting $dV_j/dz_j = 0$ (where we use this notation for full differentiation, keeping $z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_p$ constant). This allows us to show that $\mathbf{z}_0$ satisfies the first order condition. The second order condition is that the Hessian $d^2V_j/dz_j^2$ be negative definite at the joint origin. If this holds for all $j$ at $\mathbf{z}_0$, then $\mathbf{z}_0$ is a LSNE. However, we need only examine this condition for the vote function $V_1$ for the lowest valence party. As we shall show, this condition on the Hessian of $V_1$ is equivalent to the condition on $C_1$, and if the condition holds for $V_1$, then the Hessians for $V_2, \ldots, V_p$ are all negative definite at $\mathbf{z}_0$. As usual, conditions on $C_1$ for the eigenvalues to be negative depend on the trace, trace($C_1$), and determinant, det($C_1$), of $C_1$. These depend on the value of $A_1$ and on the electoral variance/covariance matrix, $\frac{1}{n}\nabla$. Using the determinant of $C_1$, we can show that $2A_1v^2 < 1$ is a sufficient condition for the eigenvalues to be negative. In terms of the "convergence coefficient" $c(\boldsymbol{\lambda},\beta;\Psi)$ we can write this as $c(\boldsymbol{\lambda},\beta;\Psi) < 1$. In a policy space of dimension $w$, the necessary condition on $C_1$, induced from the condition on the Hessian of $V_1$, is that $c(\boldsymbol{\lambda},\beta;\Psi) \leq w$. This condition is obtained from examining the trace of $C_1$ . If this necessary condition for $V_1$ fails, then $\mathbf{z}_0$ can be a neither a LNE nor a LSNE.

Ceteris paribus, a LNE at the joint origin is "less likely" the greater are the parameters $\beta$, $\lambda_p - \lambda_1$ and $v^2$.

Note that for a general spatial model with an arbitrary, non-Euclidean but differentiable metric $\Pi(x_i, z_j) = ||x_i - z_j||$, a similar expression for $A_1$ can be obtained., but in this case the covariance term $\frac{1}{n}\nabla$ will not have such a ready interpretation. Note also that if the non-differentiable Cartesian metric $\Pi(x_i, z_j) = \Sigma_{k=1}^{w}|x_{ik} - z_{jk}|$ were used, then the first order condition would be satisfied at the median rather than the mean.

Even when the sufficient condition is satisfied, so the joint origin is an LSNE, the concavity condition (equivalent to the negative semi definiteness of all Hessians *everywhere* ) is so strong that there is no good reason to expect it to hold. The empirical analyses of Israel , presented below, show that the necessary condition fails. In this polity, a PNE , even if it exists,will generally not occur at the origin.

The Theorem immediately gives the folowing Corollaries.

**Corollary 1.**In the case that $X$ is $w$-dimensional. then the sufficient condition for the joint origin to be a LSNE for the model $M(\boldsymbol{\lambda},\beta;\Psi)$is that $c(\boldsymbol{\lambda},\beta;\Psi) < 1$, while the necessary condition for the joint origin to be a LNE is that $c(\boldsymbol{\lambda},\beta;\Psi) \leq w$.

**Corollary 2.** In the two dimensional case, the two eigenvalues of $C_1$ for the model $M(\boldsymbol{\lambda},\beta;\Psi)$ are

$$
\begin{aligned}
a_1 &= A_1\{[v_1^2 + v_2^2] + [[v_1^2 - v_2^2]^2 + 4(v_1, v_2)^2]^{\frac{1}{2}}\} - 1 \\
a_2 &= A_1\{[v_1^2 + v_2^2] - [[v_1^2 - v_2^2]^2 + 4(v_1, v_2)^2]^{\frac{1}{2}}\} - 1.
\end{aligned}
$$

Train (2003, p.39) comments that the "difference between extreme value and independent normal errors is indistinguishable empirically". For this reason, in examining whether convergence can be expected in the empirical logit model, we use the result for the formal model, $M(\boldsymbol{\lambda},\beta;\Psi)$.Obviously Corollaries 1 and 2 can be used to determine the eigenvalues of the appropriate Hessians for the various models.

Recent work by Banks and Duggan (2005) has examined two party competition for the probabilistic vote model. Instead of vote maximization, they assume each party $j$ attempts to maximize the *plurality function* $U_j(z_j, z_k) = V_j(z_j, z_k) - V_k(z_j, z_k)$.To demonstrate that the joint mean $(x^*, x^*)$ is a PNE of the plurality maximization game they use the concavity of the plurality vote functions. It is obvious however that if the eigenvalues of the Hessians just considered are not all non-positive, then concavity will fail. Analogues of Theorem 1 and the corollaries can then can be developed to obtain conditions for existense of PNE in the plurality two party game, depending on the distribution assumptions on the errors.

In the next seection we apply this model to elections in the Netherlands and Israel.

# 3 Empirical Analyses

## 3.1 The vote maximizing model in the Netherlands.

We consider a logit model for the elections of 1977 and 1981 in the Netherlands (Schofield, Martin, Quinn and Whitford, 1998, and Quinn, Martin and Whitford, 1999). There are four main parties : Labor (PvdA),Christain Democratic Appeal (CDA), Liberals (VVD) and Democrats (D66), with approximately 40%, 35%,20% and 5% of the popular vote.

Figure 1 gives the estimated positions of the parties and the electoral distribution circa 1980, while Table 2 gives data on the elctions of 1979 and 1981.

For empirical analysis we include sociodemographic variables($SD$) such as education, religion etc. The characterisics of individual $i$ are given by the vector $\eta_i$, while the effect of these is given by the transposed vector $\theta_j^{\mathrm{T}}$. Thus we change the voter utility from (1) to

$$u_{ij}(x_i, z_j) = \lambda_{ij} - \beta\|x_i - z_j\|^2 + \theta_j^{\mathrm{T}}\eta_i. \tag{9}$$

The MNL model with SD we denote $M(\boldsymbol{\lambda},\beta,\theta;\Psi)$.

The convergence coefficients eigenvalues and log marginal likelihoods (LML) for the two MNL models are given in Table 2.

The Bayes' Factor ( or difference in LML) is clearly significant when valence terms are added. Thus adding valence to the MNL model without SD has a Bayes factor of 75=[-531-(-606)], while the Bayes factor for adding valence to the MNL with SD is 101=464-(-565)].

We can illustrate the computation of these parameters as follows. As Figure 1 indicates the electoral variance on the first axis is $v_1^2 = 0.658$, while on the second it is $v_2^2 = 0.289$. The covariance is neglible.

Using Corollary 2 we can compute the eigenvalues for the MNL estimation for the model $M(\boldsymbol{\lambda},\beta,\theta;\Psi)$with SD.. At the origin the probability of voting for the D66 is

$$\begin{aligned}
\rho_{d66} &= \frac{1}{1+e^{1.596}+e^{1.403}+e^{1.015}} = 0.074. \\
\text{Thus } A_{d66} &= 0.737(0.852) = 0.627. \\
C_{d66} &= (1.25)\begin{pmatrix} 0.658 & 0 \\ 0 & 0.289 \end{pmatrix} - I = \begin{pmatrix} -0.18 & 0 \\ 0 & -0.64 \end{pmatrix} \\
c(\Psi) &= 1.187
\end{aligned}$$

For this model, both eigenvalues are negative, so the origin is a local equilibrium for the vote maximising game.

This conflict between the convergence implied by the theorem, and the divergent postions seen in Figure 1 suggests that the CDA positioned itself off the first economic or principal electoral axis, in order so as to be better positioned with regard to coalition outcomes . We can illustrate this by the following example of "coalition risk".

### 3.1.1 Coalition risk

As Table 1 indicates ,with uncertainty about the elections,there are two probable coalition structures:

$$\mathbb{D}_0 = \{\text{PvdA,CDA}\},\{\text{PvdA,VVD}\},\{\text{CDA,VVD}\}$$
$$\mathbb{D}_1 = \{\text{PvdA,CDA}\},\{\text{PvdA,VVD,D66}\},\{\text{CDA,VVD,D66}\}.$$

The second structure is denoted $\mathbb{D}_1$ because it is evident that a "structurally stable policy core" can occur at a profile $\mathbf{z}=(z_{PvdA}, z_{CDA}, z_{VVD}, z_{D66})$ whenever $z_{PvdA}$ lies in the interior of the convex hull of the three positions $z_{CDA}, z_{VVD}, z_{D66}$. To see this note that although {CDA,VVD,D66} is a decisive coalition , its members cannot agree over a policy position that they all prefer to $z_{PvdA}$. It is also the case that this situation is insensitive to small perturbations of party positions, and so the core at $z_{PvdA}$ is structurally stable, or *ss* ( see Laver and Schofield, 1990, for the definition of this term). We denote the structurally stable core at the vector **z** and the coalition structure $\mathbb{D}_1$by $\mathbb{SC}_1(\mathbf{z})$and call PvdA a *core party.* Laver and Schofield (1990) argue that the core party can construct a minority governmrnt and control all perquisites.

Because it is common we shall typically use the notation $\mathbb{D}_{party}$ for the family of coalition structures that admit a *ss* core at the position of some *party* , where *party*refers to the largest party able to position itself at a structurally stable core position.We shall also say that *party* is *dominant* under any coalition structure in $\mathbb{D}_{party}$ and profile **z**.

On the other hand, with the decisive structure $\mathbb{D}_0$ there is no vector of party positions that gives a structurally stable core outcome.

To develop a model,of coaltion positioning, consider the question of optimal positions in a situation where the probability $\pi_1^*(\mathbf{z})$ of the coalition structure $\mathbb{D}_1$ can be assumed to be **0** for all feasible vectors **z**. Ignoring governmrnt perquisites are suppose the positions of the party leaders of the three parties are given ,as in Figure 2 by $\mathbf{z} = (z_{PvdA}, z_{VVD}, z_{CDA}) = ((-\checkmark 3, 0), (\checkmark 3, 0), (0, 1)$. Assume further that the outcome associated with any vector **z** of party positions and the coalition structure $\mathbb{D}_0$ will lie within the convex hull of the party positions.For purposes of illustration,let us assume that at the profile **z,** the beliefs of the party leaders over coalition outcomes are given by a lottery, $\tilde{g}_0^0(\mathbf{z})$ that specifies the uniform distribution across the convex hull of the vectors that comprise $\mathbf{z}$ To show why the best response of the CDA ,may be "radical" suppose the positions of PvdA and VVD are given by $(z_{PvdA}, z_{VVD})$ as in Figure 2. and let us compare the utilities for the CDA at the positions $z_{CDA}^* = (0,3)$ and $z_{CDA} = (0,1)$. From the symmetry of the figure it follows that the von Neumann-Morgenstern utility function $U_{CDA}$ satisfies the equation

$$\begin{aligned} U_{CDA}(\tilde{g}_0^0(z_{PvdA}, z_{CDA}^*, z_{VVD}) &= \frac{1}{3}U_{CDA}(\tilde{g}_0^0(z_{PvdA}, z_{CDA}, z_{VVD}) + \\ &\quad \frac{1}{3}U_{CDA}(\tilde{g}_0^0(z_{PvdA}, z_{CDA}, z_{CDA}^*) + \\ &\quad \frac{1}{3}U_{CDA}(\tilde{g}_0^0(z_{CDA}^*, z_{CDA}, z_{VVD}) \\ &= U_{CDA}(\tilde{g}_0^0(z_{PvdA}, z_{CDA}, z_{VVD}) \end{aligned}$$

By continuity, there is a position denoted $y_{CDA}$ on the arc [(0,1),(0,3)] which gives the best response of the CDA to $(z_{PvdA}, z_{VVD})$. The analysis of the general case of coalition risk is developed further in Schofield and Parks, 2000). Assuming that policy leaders have utility functions that involve a Euclidean metric on policy distance they show that non centrist LSNE can occur for the profile $(U_{CDA}.U_{VVD}, U_{PvdA})$.

This example suggests that members of a party may choose positions for their leaders in order to infuence coalition bargaining in their favor. We may call this phenomenon *the centrifugal effect of coalition risk.*

### 3.2 The Israel Knesset in 1996.

To further illustrate the theory, consider the case of Israel in 1996. Figure 3 shows the estimated positions of the parties at the time of the 1996 election while Table 3 presents election results for 1988-2003. Just as with the above example, we can readily show that both eigenvalues of the NRP are positive. Indeed it is obvious that there is a principal component of the electoral distribution, and this axis is the eigenspace of the major eigenvalue. The formal analysis indicates that low valence parties should position themselves on this eigenspace as illustrated in the simulation given below in Figure 4.The fact that Shas is not located on this axis in Figure 3 suggests that it is responding to coalition risk.

In 1996, and using the model $M(\boldsymbol{\lambda},\beta,\theta;\Psi$ the lowest valence party was the NRP with valence –4.52. The spatial coefficient was $\beta = 1.12$, and the electoral variances on the two axes were 1.0 and 0.732 respectively. We compute

$$\begin{aligned}
\rho_{NRP} &\simeq \frac{1}{1+e^{4.15+4.52}+e^{3.14+4.52}} \simeq 0. \\
\text{Thus } A_{NRP} &= \beta = 1.12. \\
C_{NRP} &= 2(1.12)\begin{pmatrix} 1.0 & 0.591 \\ 0.591 & 0.732 \end{pmatrix} - I = \begin{pmatrix} 1.24 & 1.32 \\ 1.32 & 0.64 \end{pmatrix} \\
c(\Psi) &= 3.88
\end{aligned}$$

Then the eigenvalues are 2.28 and -0.40, giving a saddlepoint, and a value for the convergence coefficient of 3.88. The major eigenvector for the NRP is (1.0,0.8), and along this axis the NRP vote share function increases as the party moves away from the origin. The minor, perpendicular axis is given by the vector (1,-1.25) and on this axis the NRP vote share decreases. Figure 4, gives one of the local equilibria in 1996, obtained by simulation of the model..The figure makes it clear that the vote maximizing positions lie on the principal axis through the origin and the point (1.0,0.8). Five different LSNE were located, in all cases, the two high valence parties, Labor and Likud, were located at almost precisely the same positions. The only difference between the various equilibria were that the positions of the low valence parties were perturbations of one other.The minor, perpendicular axis is given by the vector (1,-1.25) and on this axis the NRP vote share decreases.

Figure 5 shows the estimated party positions and voter distribution in 1992, while Figure 6 shows the simulated LSNE positions. Again the formal model predicts that a low valence party such as Shas should adopt a position on the principal electoral axis.

The simulations of vote maximizing positions were compatible with the predictions of the formal model based on the extreme value distribution. For both 1992 and 1996, all parties were able to increase vote shares by moving away from the origin, along the principal axis, as determined by the large , positive principal eigenvalue . In particular, the simulation confirms the logic of the above analysis. Low valence parties, such as the NRP and Shas, in order to maximize vote shares must move far from the electoral center . Their optimal positions will lie either in the "north east" quadrant or the "south west" quadrant The vote maximizing model, without any additionnal information, cannot determine which way the low valence parties will move. Indeed, the simulations of the empirical model found multiple LSNE essentially differing only in permutations of the low valence party positions.

In contrast, since the valence difference between Labor and Likud was relatively low in all three elections, the relevant eigenvalues on the major axis, for their Hessians at the origin, were also low (but still positive), and their optimal positions would be relatively close to, but not identical to, the electoral mean. The simulation figures for all three elections are therefore compatible with this theoretical inference. The figures also suggest that every party, in local equilibrium, should adopt a position that maintained a minimum distance from every other party. The formal analysis as well as the simulation exercise, suggests that this minimum distance depends on the valences of the neighboring parties. Intuitively it is clear that once the low valence parties vacate the origin, then high valence parties, like Likud and Labor will position themselves almost symmetrically about the origin, and along the major axis. It should be noted that the positions of Labor and Likud are particularly closely matched by their positions in the simulated vote maximizing equilibria.

Clearly, the configuration of equilibrium party positions will fluctuate as the valences of the large parties change in response to exogenous shocks. The logic of the model remains valid however, since the low valence parties will be obliged to adopt relatively "radical" positions in order to maximize their vote shares.

It is important to note however, that the position of Shas in Figure 3 is not at all similar to its estimated vote maximizing equilibrium position as given in Figure 4. Indeed all simulated vote maximizing equilibrium positions for Shas were on the principal axis. This suggests that Shas adopted a position off the principal axis so as to be able to pivot between the coalitions led bt either Labor or Likud. In fact Shas was a coalition ally of Likud until very recently This strongly suggets that an appropriate model of party positioning assumes that parties are concerned with policy, and adopt positions with a view towards the coalitions that may form after the elction. One way to wxpress this inference is as follows.

*Conclusion 1.*The similarity between the simulation of LSNE for the empirical model and the results of the formal analysis indicate that the formal model can be used to infer the unknown motivations of party principals.

*Conclusion 2.* The close correspondence between the simulated LSNE based on the empirical analysis and the estimated actual political configuration suggests that the true utility function for party each party $j$ has the form $U_j(\mathbf{z}) = V_j(\mathbf{z}) + \delta_j(\mathbf{z})$, where $\delta_j(\mathbf{z})$ may depend on the beliefs of party members about the post election coalition possibilities.

To extend the above model, we now introduce the notion of electoral uncertainty.

# 4 Local Equilibria under Electoral Uncertainty

Using the expected vote share functions as the maximand for the electoral game has its attraction. As we have seen , the expected vote share functions can be readiliy computed because they are linear functions of the entries in the voter probability matrix $(\rho_{ij}(\mathbf{z}))$. At least for two party competition, more natural payoff functions to use are the partys' *probability of victory.* To develop this idea, we can introduce the idea of the stochastic vote share functions $\{V_j^*(\mathbf{z}) : j = 1, ...p\}$.Then the expected vote share functions used above are simply the expectations $\{Exp(V_j^*(\mathbf{z})\}$ of these stochastic variables. In the two party case, the probability of victory for agents 1 and 2 can be written

$$\pi_1(\mathbf{z}) = \Pr[V_1^*(\mathbf{z}) > V_2^*(\mathbf{z})] \text{ and } \pi_2(\mathbf{z}) = \Pr[V_2^*(\mathbf{z}) > V_1^*(\mathbf{z})]$$

As Patty(2004a) has commented, an agent's probability of victory is a complicated nonlinear expression of the voters' behavior as described by the vote matrix $(\rho_{ij}(\mathbf{z}))$ . Just as we can define LNE and PNE for the game given by the profile function $V : X^p \to \mathbb{R}^p$, we can also define LNE and PNE for the two party profile function $\pi = (\pi_1, \pi_2) : X^2 \to \mathbb{R}^2$ . Duggan (2000) and Patty(2004a) have explored those conditions under which equilibria for expected vote share functions and probability of victory are identical. As might be expected these equilibria are generically different (Patty,2004b).

We shall now develop a model based on electoral uncertainty, and can be considered to be a generalization of the Duggan/ Patty models of two-party competition. To do this we introduce the idea of a party principal.

The strategy, $z_j$,of party $j$ corresponds to the position of the party leader and is chosen by the *party principal*, $j$, whose prefereed position is $x_j$.We shall develop the model first with only two parties.If party $j$ wins the election with a leader at position $z_j \in X$, while party $j$ receives a non-policy perquisite $\delta_j$, then the payoff to the principal , $j$, is

$$U_j^{\alpha_j}(z_j, \delta_j) = - \parallel z_j - \chi_j \parallel^2 + \alpha_j \delta_j$$

Thus the profile function $U = (U_1, U_2) : X^2 \to \mathbb{R}^2$ can be taken to be given by the expected payoffs

$$\begin{aligned} U_1(z_1, z_2) &= \pi_1(z_1, z_2) U_1^{\alpha_1} U_1(z_1, \delta_1) + \pi_2(z_1, z_2) U_1^{\alpha_1}(z_2, 0) \\ U_2(z_1, z_2) &= \pi_2(z_1, z_2) U_2^{\alpha_2}(z_2, \delta_2) + \pi_1(z_1, z_2) U_2^{\alpha_2}(z_1, 0) \end{aligned}$$

This expression ignores the probability of a draw. In the case of a draw, the outcome can be assumed to be lottery between the party positions $z_1$ and $z_2$.The multiparty model we propose is a natural extension of the two party model and is built as follows.As before, we can examine conditions sufficient for existence of LNE or PNE for for such a two party profile function (See Cox (1984) for an example). To extend this to a model of *multiparty* competition ( with $p \geq 3$, we must deal with the fact that it is possible for no party gains a majority of the Parliamentary seats (or in the case of US Presidential elections, a majority of the electoral college) . We shall argue that in multiparty competition the possible outcomes of the election correspond to the family of all decisive coalition structures

$$\mathbb{D} = \{\mathcal{D}_1, \ldots \mathcal{D}_t, ..\mathcal{D}_T\}$$

which can be obtained from the set $P$ of parties. For convenience we may assume that the subfamily $\{\mathcal{D}_1, \ldots \mathcal{D}_p\}$, with $p < T$, correspond to the subfamily of coalition structures where the parties $\{1, ..p\}$,respectively,win the election with a majority of the seats in the Parliament. Notice that the outcomes $\{\mathcal{D}_1, \ldots \mathcal{D}_T\}$ are defined in terms of the distribution of seat shares $(S_1, S_2, ...S_p)$ in the Parliament, and not simply vote shares. The more interesting cases are given by $t > p$, and for convenience we can assume that for such a $t$, the coalition structure $\mathcal{D}_t = \{M \subset N : \Sigma_{j \in M} S_j > 1/2\}$. Decisive coalition structures can of course be defined in more complex ways. Since there is an intrinsic uncertainty in the way votes are translated into seats, it makes sense to focus on the probabilities associated with these decisive structures. At a vector $\mathbf{z}$ of positions of party leaders, the probability that $\mathcal{D}_t$ occurs is denoted $\pi_t(\mathbf{z})$. We can also assume that the vector

$$(\pi_1(\mathbf{z}), ...\pi_p(\mathbf{z}))$$

corresponds to the probabilities that parties 1,..$p$ ,respectively,win the election. When party $j$ wins then the outcome, of course, is the situation $(z_j, 1)$.That is party $j$ implements the position $z_j$ of its party leader and takes a share 1 of non-policy perquisites. When no party wins, but a decisive coalition $\mathcal{D}_t$ occurs, for $t \geq p+1$, then the outcome is a lottery which we denote by $\tilde{g}_t^{\alpha}(\mathbf{z})$ . We assume $\tilde{g}_t^{\alpha}(\mathbf{z}). \in \tilde{W} = Bor(X \times \Delta_P)$.Here $\Delta_P$ is the set of possible distributions of government perquisites among the parties, and $W = (X \times \Delta_P)$ while $(Bor(X \times \Delta_P)$ is the space of Borel probability measures over $X \times \Delta_P$ endowed with the weak topology (Parthasarathy,1967). Thus $\tilde{g}_t^{\alpha}(\mathbf{z})$ specifies a finite lottery of points in $X$ coupled with a lottery of distributions of perquisites among the parties belonging to the decisive structure $\mathcal{D}_t$ (See Banks and Duggan, 2000) for a method of deriving this lottery). We implicitly assume that the utility function of the principal of party $j$, given by the expression $U_j(\chi_j, z_j, \delta_j)$ above, can be regarded as a function $U_j : (X \times \ \Delta_P) \to \mathbb{R}$ and can be extended to a function $U_j : (Bor(X \times \ \Delta_P) \to \mathbb{R}$, measurable with respect to the sigma-algebra on $Bor(X \times \ \Delta_P)$.Note that if $g \in \tilde{W}$, then it is a measure on the Borel *sigma-algebra* of $W$. Since $U_j \ : \ W \to \mathbb{R}$ is assumed measurable the integral $\int U_j \ dg$ is well defined and can be identified with $U_j(g) \in \mathbb{R}$. In the weak topology a sequence $\{g_k\}$ of measures converges to $g$ if and only if $\int U \ \ dg_k$ converges to $\int U \ dg$ for *every* bounded, continuous utility function $U$ with domain $W$. We further assume that $\tilde{g}_t^{\alpha} \ : X^P \to \tilde{W}$ is $C^2-$differentiable as well as continuous .This means that for all $j$ the induced function $U_j^t \ : X^P \to \mathbb{R}$ ,given by $U_j^t(\mathbf{z}) \ = \ U_j(\tilde{g}_t^{\alpha}(\mathbf{z})$, is also $C^2-$differentiable, so its Hessian with respect to $z_j$ is everywhere defined and continuous. Observe that $\tilde{g}_t^{\alpha}$ is used to model the common beliefs of the principals concerning the outcome of political bargaining in the post election situation given by $\mathcal{D}_t$. The common beliefs of the principals concerning electoral outcomes are given by a $C^2-$differentiable function $\pi : X^p \to \Delta_T$ from $X^p$ to the simplex $\Delta \ _T$ (of dimension *T-1*) where $T$ is the cardinality of the set of all possible coalition structures. At a vector $\mathbf{z}$ of positions of party leaders, the probability is $\pi_t(\mathbf{z}\ )$ that the distribution of parliamentary seats among the parties gives the decisive structure $\mathcal{D}_t$ . The *electoral probability function* $\pi$ models the uncertainty associated with the election. Note that this uncertainty also includes the uncertainty over the valences of the various party leaders. We now provide the formal definitions for the multiparty political game.

**Definition 6: The game form derived from policy preferences**.

**(i)** The *electoral probability function* $\pi = (\pi_1, .., \pi_T) : X^p \to \Delta_T$ is a smooth function from $X^p$ to the simplex $\Delta \ _T$ (of dimension *T-1*) where

$\mathbb{D} = \{\mathcal{D}_1, \ldots \mathcal{D}_T\ \}$is the set of all possible decisive coalition structures. This function captures the notion of *electoral risk.*

**(ii)** For fixed $\mathcal{D}_t$, the outcome of bargaining at the parameter $\alpha = (\alpha_1, .., \alpha_p)$ and at the strategy vector $\mathbf{z}$ is a lottery $\tilde{g}_t^{\alpha}(\mathbf{z}) \ \in (Bor(X \times \Delta_P)$.This caaptures the notion of *coalition risk at* $\mathcal{D}_t$.

**(iii)** At the fixed decisive structure ,$\mathcal{D}_t$, and strategy vector $\mathbf{z}$,the payoff to the principal of party $j$ is

$$U_j^t(\mathbf{z}) \ = \ U_j(\tilde{g}_t^{\alpha}(\mathbf{z}))$$

**(iv)** The *game form* $\{\tilde{g}_t^{\alpha}, \pi_t\}$at the parameter $\alpha$ is denoted $\tilde{g}^{\alpha}$. At the strategy vector $\mathbf{z}$,the payoff to the principal $j$ is given by the von Neumann-Morgenstern utility function

$$U_j^g(\mathbf{z}) \;=\; U_j(\tilde{g}^\alpha(\mathbf{z})) \;=\; \sum_{t=1,..T} \pi_t(\mathbf{z})\; U_j^t(\mathbf{z}).$$

(**v**)The *game profile* derived from the game form $\tilde{g}^\alpha$ at the utility profile $\{ U_j\}$ is denoted

$$U^g = (U_1 \circ \tilde{g}^\alpha, ...\; U_p \circ \tilde{g}^\alpha) = (..U_j^g..) : X^p \;\to \mathbb{R}^p$$

(**vi**) The game form $\tilde{g}^\alpha$ is *smooth* iff the function $U^g : X^p \;\to \mathbb{R}^p$ is $C^2$ $-$differentiable. Let $\mathbb{U}(X^p\;, \mathbb{R}^p)$ be the set of $C^2-$differentiable utility profiles $\{U : X^p \;\to \mathbb{R}^p\}$ endowed with the $C^2$ topology. (Roughly speaking, two profiles are close in this topology if all values and first and second derivatives of each $U_j$ are close).

(**vii**) A *generic property in* $\mathbb{U}(X^p\;, \mathbb{R}^p)$ is one that is true for a set of profiles which is open dense in the $C^2$ topology (See Hirsch 1984 and Schofield, 2003 for the definition of the $C^2-$ topology and the notion of generic property)

(**viii**) For the fixed smooth game form $\tilde{g}^\alpha$, let $\{U^g : X^p \;\to \mathbb{R}^p\} \subset \mathbb{U}(X^p\;, \mathbb{R}^p)$ be the set of utility profiles induced as the parameters of voter ideal points and electoral beliefs are allowed to vary.

(**ix**) Let $\mathbb{G}$ be the set of smooth game forms. The transformation $\tilde{g} \to U^g : \mathbb{G} \to \mathbb{U}(X^p\;, \mathbb{R}^p)$induces a topology on the set $\mathbb{G}$ , where this topology is obtained by taking the coarsest topology such that this transformation is continuous.

(**x**) The vector $\mathbf{z}^*$=$(z_1^*, ...z_{j-1}^*, z_j^*, z_{j+1}^*..z_p^*) \in X^p$ is a *local strict Nash equilibrium* (LSNE) for the profile $U \in \mathbb{U}(X^p\;, \mathbb{R}^p)$ iff for each $j$ there is a neighborhood $X_j$ of $z_j^*$ in $X$, with the property that

$$U_j(z_1^*, \ldots, z_j^*, z_{j+1}^*, \ldots, z_p^*) \;>\; U_j(z_1^*, \ldots, z_j, z_{i+1}^*, \ldots, z_p^*)$$

for all $z_j \in X_j - \{z_j^*\}$.

(**xi**) $\mathbf{z}^* \in X^p$ is a *critical Nash equilibrium* (CNE) for the profile $U$ iff, for each $j$,the first order condition $\frac{dU_j}{dz_j} \;=\; 0$ is satisfied at $\mathbf{z}^*$ .

(**xii**) A *strict Nash Equilibrium* (PSNE) for $U$ is a LSNE for $U$ with the additional requirement that each $X_j$ is in fact $X$.

(**xii**) For a fixed profile $x \in X^n$ of voter ideal points ,fixed electoral beliefs $\pi$, and fixed game form $g$, the vector $\mathbf{z}^*$is called the LSNE, PSNE or CNE if it satisfies the appropriate condition for the game profile $U^g : X^p \;\to \mathbb{R}^p$.

**xiv)** An LSNE $\mathbf{z}^* \in X^p$ for the profile $U$ is *locally isolated* iff there is a neighborhood $Z^*$ of $\mathbf{z}^*$ in $X^p$ which contains no LSNE for $U$ other than $\mathbf{z}^*$.

Schofield(2001) and Schofield and Sened(2002) show that ,for each parameter ,$\alpha$,there is an open dense set of smooth game forms,with the property that each form $\tilde{g}^\alpha$ in the set exhibits a LSNE. In principle, this result suggests that if the electoral function is smooth, and if the outcome of coalition bargaining is differentiable in the location of parties, then there will exist local equilibria which can be used to deduce party positions. Of course, this model is very much more complex than the vote maximizing version presented in the previous section.

For the Theorem to valid, we require that the strategy space $X^p$ is compact convex subset of a finite dimensional topological vector space. Such a space we shall call a *Fan space* (Fan, 1964) We also require the following boundary condition on the profile. Say a profile $U \in \mathbb{U}(X^p, \mathbb{R}^p)$ satisfies the *boundary condition* if for every point $\mathbf{z}$ on the boundary of the Fan space, $X^p$, the induced gradient ( $\frac{dU_1}{dz_1}$ , ..$\frac{dU_p}{dz_p}$ ) points towards the interior of $X^p$.Let $\mathbb{U}_b(X^p, \mathbb{R}^p)$ be the subspace of profiles satisfying the boundary condition.

**Theorem 2.** Assume $X$ is a Fan space and $p$ is finite. Then the property that the LSNE exists and is locally isolated is generic in the topological space $\mathbb{U}_b(X^p, \mathbb{R}^p)$ .

*Sketch of Proof.* For each $j$, consider the set $T_j = \{\mathbf{z} \in X^p : \frac{dU_j}{dz_j} = 0\}$.

By the inverse function theorem $T_j$ is generically a smooth manifold of dimension $(p-1)\dim(X)$. By transversality theory the intersection $\cap_{j \in P} T_j$ is of codimension $p \dim(X)$ in $X^p$. But $X^p$ has dimension $p\dim(X) = pw$. Since the set of CNE $\equiv \cap_{j \in P} T_j$, this shows that there is an open dense set $\mathbb{U}_b^*(X^p, \mathbb{R}^p)$ such that for each $U \in .\mathbb{U}_b^*(X^p, \mathbb{R}^p)$ ,the set of CNE of $U$ is of dimension 0, that is, it consists of locally isolated points. Now for each such $U$, construct a gradient field $\mu(U)$ on $X^p$ whose zeros consist precisely of the CNE of $U$ (see Schofield 1998a for this construction). Since $X$ is assumed compact, convex it is homeomorphic to the ball. Because of the boundary assumption on profiles, the field $\mu(U)$ points inward on the boundary of $X^p$. The Morse inequalities (Milnor 1963, Dierker 1976) imply that there must be at least one critical point $\mathbf{z}^*$of $\mu(U)$ whose index is maximal. Thus the Hessian of each $U_j$ at $\mathbf{z}^*$ must be negative definite , and $\mathbf{z}^*$ corresponds to a locally isolated LSNE of the profile $U$. *QED.*

This theorem suggests that if we consider any fixed game form $\tilde{g}$ , then existence of locally isolated LSNE is a generic property in the space $U^g$ : $X^p \to \mathbb{R}^p\} \subset \mathbb{U}(X^p, \mathbb{R}^p)$. Moreover, if the transformation $\mathbb{G} \to \mathbb{U}(X^p, \mathbb{R}^p)$

(from game forms to utility profiles) is well behaved, in the sense that open sets are tranformed to open sets, then continuity of the transformation would imply that existence of LSNE is a generic property in the space $\mathbb{G}$

### 4.1 An example of coalition risk in Israel.

To see the coalition effect. we note that, after the 1992 election in Israel, the coalition $M_1 = \{$Labor, Meretz, ADL, Communist Party$\}$ controlled 61 seats while the coalition ,$M_2$ of the remaining parties , including Likud controlled only 59 seats out of 120. Thus the 1992 decisive structure may be written $\mathcal{D}_{1992}$ and has the form $\{ M_1, M_2\cup$ Labor,$M_2\cup$ Meretz,$\}$.Since the Labor position $z_{labor}$ in Figure 5 obviously lies inside the convex hull of the set of positions of parties in any winning coalition, we observe that $z_{labor} = \mathbb{SC}_1(\mathbf{z})$ is also the structurally stable core.. Now it is possible to find a profile $z$ with $z_{likud}$ lying inside the convex hull of the positions of the parties in $M_1$. Such a profile we regard as empirically infeasible. It therefore follows that Labor would be the uniquely feasible core party under $\mathcal{D}_{1992}$ .Thus $\mathcal{D}_{1992} \in \mathbb{D}_{labor}$ . Moreover Labor is *dominant* under $\mathcal{D}_{1992}$ with the party positions similar to those given in Figure 3. As above we refer to this family of coalition structures as $\mathbb{D}_1$.

Again, using Table 3, we note that, after the 1996 election the coalition ,$M_2$ controlled 68 seats and so belonged to $\mathcal{D}_{1996}$.Clearly there is a profile $\mathbf{z}$ with $z_{labor}$ lying inside the convex hull of the positions of the parties in $M_2$ ,but again this can be regarded as infeasible .We can therefore assert that there is no feasible $z$ such that $\mathbb{SC}^0_{1996}(\mathbf{z})$ is non-empty, which leads us to infer that $\mathcal{D}_{1996} \in \mathbb{D}_0$.

Prior to the 1996 election there were therefore two qualitatively distinct possible outcomes, namely $\{\mathbb{D}_0$ ,$\mathbb{D}_1\}$. To examine optimal party positions prior to the election of 1996, first consider the outcomes under $\mathbb{D}_1$ . Without perquisites the outcome will be $\mathbb{SC}_1(\mathbf{z}) = z_{labor.}$ Since we assume party principals have policy preferences, the principal of Likud should choose a position to minimize $\pi^*_1(\mathbf{z}) = \Pr[\mathbb{D}_1]$. One obvious way to do this is to choose $z_{likud}$ as a best reponse in order to maximize its expected vote share

Now consider the situation under $\mathbb{D}_0$. We assume that the government policy position lies inside the "heart" -that is a a subset of the convex hull of the positions in the coalition $M_3 = \{$Likud,Labor,Shas$\}$. As in the previous example from the Netherlands, this suggests that Shas adopt a "radical" position in order to influence coalition outcomes.

To summarise: Labor should adopt a position as a best response in order to maximize $\pi^*_1(\mathbf{z})$ while Likud should minimize $\pi^*_1(\mathbf{z})$ . As a first approximation, these strategies can be interpreted as maximizing the vote share functions $V_{labor}, V_{likud}$ respectively. For Shas, and other small religious parties, optimal strategies will depend on their estimates of $\pi_0^*$ and $\pi_1^*$. Ceteris paribus, the larger is $\pi_0^*(z)$ the further will the optimal Shas position be from the axis drawn between the Labor and Likud . Comparison of the estimated party positions in Figure 3 and the simulated vote maximizing positions in Figure 4 suggests that the position of Shas in Figure 3 is compatible with this interpretation of the motivations of the party principals.

## 4.2 Illustration of Party Switching in the Israel Knesset in 2005-6.

We may also make some comments with regard to recent changes in Israel. Figure 7 provides a schematic representation of the current Knesset based on the party positions estimated in Figure 3 and seat allocations given in Table 3. The figure shows Labor with 21 seats, after Am Ehad, with 2 seats, joined Labor in 2003, while Likud has 40 seats after being joined by Olim, with 2 seats. Although Barak, of Labor, became Prime Minister in 1999, he was defeated by Ariel Sharon, of Likud, in the election for prime minister in 2000. The set denoted the heart in this figure represents the coalition possibilities open to Sharon.

The figure can be used to understand the consequences after Sharon seemingly changed his policy on the security issue in August, 2005, by pulling out of the Gaza Strip. First, the Likud party elite reacted strongly against this change in policy. In the first week of November, 2005, Amir Peretz, a union activist, and leader of Am Ehad, won the election for leader of the Labor Party. This event can be seen as an illustration of the argument given in Schofield (2005a,b) that a party that fails to attract voters because of a low relative valence will eventually be controlled by party activists. From this perspective, the low valence of Labor vis-à-vis Sharon was the reason the Labor members chose Peretz.

Many observers regarded the change in the leadership of Labor as a critical transformation in the political map of Israel. However, as Figure 8 suggests, the shift to the left by Labor under Peretz had no effect on the heart of the Knesset. According to the model, there would be no effect on party bargaining.

However, the move by Labor did have indirect consequence. In a high highly publisized move, Sharon left the Likud Party and signaled a strong move to the left by allying with Shimon Peres, the former leader of Labor and the author of the Oslo accords, together with a number of other senior

Labor Party members, to form the new party, *Kadima* ("Forward"). This move positioned Sharon at the origin of the electoral space at (0,0) as shown in Figure 9. By moving Labor to the left, Peretz created the opportunity for Sharon to out maneuver him. Sharon could strategically move to a position that would increase the probability that he would control the core. Because Sharon's own party members would not support him in this move, he had to leave Likud and form Kadima.

However, Figure 9 suggests that the seat strengths were insufficient for Sharon to actually conrol the core. Indeed, the heart is bounded by the three median lines drawn in the figure. Since these lines do not intersect, the core is empty. Note however, that the new configuration of the heart suggests a possible government coalition of Kadima with supporters from factions of either Likud or Labor.

In a much less publicized move, Sharon took his political maneuvering a step further, obtaining the support of Uriel Reichman, founder of the Shinui party, for Kadima. On the face of it, this move seemed hard to explain. Although Reichman is a notable figure in Israel (currently the President of IDC, the largest and most successful private university in Israel), he has never held an elected office. In fact, Sharon promised Reichman the position of Minister of Education in a Kadima coalition government. The purpose of this contract is clear from Figure 10. By obtaining Reichman's support, Sharon made a small move "south" in the the policy space towards the the structurally stable core. Indeed this position is very close to the position previously held by the Labor Party, under the leadership of Rabin, at the 1992 election in Israel.

Because of Sharon's stroke in January, 2006, Ehud Olmert, previously of Likud, became the leader of Kadima, and faced Benjamin Netanyahu, the new leader of Likud, in the elections of March 28, 2006. It is worth quoting Olmert's recent remarks about his understanding of the election:

> For 32 years, I have served the State of Israel. From the position I have assumed due to Prime Minister Sharon's illness, I see an Israel fighting difficulties and great hardships, but I also see the glimmer of hope in the eyes of many Israelis, for the first time in many years. And as Ariel Sharon said: "We must not let this new spirit, which grants our peoples hope, pass us by and leave us empty-handed - I have no intention of missing this opportunity.

Figure 10 gives an estimate of party positions at the March 28, 2006, election to the Knesset. Although Olmert's valence was obviously lower than that of Sharon, Kadima was still able to take 29 seats. Likud, together with religious parties, took 50 seats. One surprise of the election was the appearance of a pensioners' party with 7 seats. However, this had no effect on coalition bargaining. Because a coalition beteen Labour and the religious parties is infeasible, we can infer that Kadima is located at the structurally stable core position (as indicated in Figure 10). It appears that Sharon's change of policy has led to a fundamental transformation in the political configuration, from the $\mathbb{D}_0$ -coalition structure that had persisted since 1996, to a $\mathbb{D}_1-$structure associated with the new core party, Kadima. On April, 30, 2006 Olmert put together a coalition government with Labor, the Pensioners and Shas, controlling 67 seats.

## 5 Conclusions about Party Switching.

It is now possible to draw some general conclusions about party switching and pre-election agreements from the formal models presented above.

(i) Under proportional rule, and with exogeneous valence alone,there will be a centrifugal tendency associated with low valence parties which may make them disagreeable partners to high valence parties. This follows because the addition of a low valence party will surely lower the valence of the higher valence party. If activists are relevant, then their influence would be to pull the party away from the center.

(ii) Two low valence parties may combine, but this will have little effect on their combined valence, and their optimal position will still be radical. The coalition effect may be pronounced however. Neither party may be pivotal, but by combining they may become, like Shas in Israel, influential in coalition formation.

(iii) Coalescence between high valence parties may occur. In Israel, Sharon initially formed an electoral pact with Labor. This implied change in party position induced an activist backlash that led to a new party leader for Labor. The consequent policy move to the left opened up the center to a new party, Kadima..

(iv) Because low valence parties may resent the dominance of a core party, they may adjust position, or recreate themselves in order to destroy the center party dominance. This appears to have occurred in 1992 in Italy (Mershon, 2002; Schofield, 1993, Giannetti and Sened 2004)..

(iv) It is assumed here that the position adopted by a party leader is in fact chosen by a party principal who represents the diverse policy preferences of the party elite. If the party leader has low valence, then the party principal may adopt a position near the electoral periphery. This may cause party members to vacate the party in order to move to a party whose leader has higher valence.

(v) An alternative strategy for centrist members of the party is to leave en bloc in order to adopt an isolated but centrist position. In general such new parties will have low valence, and they may be extinguished.

A more general point is the equilibrium concept adopted here is that of *local Nash equilibrium.* Equilibrium positions will be sensitive to the beliefs that sustain them It is possible therefore for small changes in beliefs about the electoral response function and the coalition lottery function $\tilde{g}^{\alpha}$ to destroy the current LNE. This could cause rapid shifts in party membership, as suggested by the recent events in Israel.

In contrast, the idea of a structurally stable LNE (or $ss$LNE) can be introduced, where this refers to the stability of the LNE under perturbations of the underlying game form and set of beliefs. The recent events in Israel suggest that a $ss$LNE may have been brought into existence as a recent of a complex pattern of leadership changes and party switching, leading to a new core configuration.

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**Table 1. Election Results in the Netherlands, 1977-1981**

| Party (acronym) | 1977 (Seats) | 1981 (Seats) |
|---|---|---|
| Labor (PvdA) | 53 | 44 |
| Democrats '66 (D66) | 8 | 17 |
| Liberals (VVD) | 28 | 26 |
| Christian Dem Appeal (CDA) | 49 | 48 |
| | (138) | (135) |
| Communists (CPN) | 2 | 3 |
| Dem '70 (D70) | 1 | 0 |
| Radicals (PPR) | 3 | 3 |
| Pacific Socialists (PSP) | – | 3 |
| Reform Federation (RPF) | – | 2 |
| Reform Pol Ass (GDV) | 1 | 1 |
| Farmers Party (BP) | 1 | 0 |
| State Reform Party (SGP) | 3 | 3 |
| | (11) | (15) |
| Total | 149 | 150 |

**Table 2.**

**Log Likelihoods and eigenvalues in the Dutch Electoral Model**

| Model | coefficient | Eigenvalue 1 | Eigenvalue 2 | LML |
|---|---|---|---|---|
| MNL without valence or SD | na | na | na | -606 |
| MNL without valence, with SD | na | na | na | -565 |
| MNL with valence, no SD | 1.19 | -0.18 | -0.64 | -531 |
| MNL model with valence and SD | 1.38 | -0.04 | -0.58 | -464 |
| | | | | |

**Table 3. Elections in Israel, 1998-2003**

| | Party | *Knesset Seats* 1988 | 1992 | 1996 | 1999 | 2003 |
|---|---|---|---|---|---|---|
| **Left** | Labor(LAB) | 39 | 44 | 34 | 28 | 19[a] |
| | Democrat (ADL) | 1 | 2 | 4 | 5 | 2[a] |
| | Meretz | -- | 12 | 9 | 10 | 6 |
| | CRM, MPM,PLP | 9 | -- | -- | -- | 3 |
| | Communist (HS) | 4 | 3 | 5 | 3 | 3 |
| | Balad | -- | -- | -- | 2 | 3 |
| **Subtotal** | | *53* | *61* | *52* | *48* | *36* |
| **Center** | Olim | -- | -- | 7 | 6 | 2[b] |
| | III Way | -- | -- | 4 | -- | -- |
| | Center | -- | -- | -- | 6 | -- |
| | Shinui (S) | 2 | -- | -- | 6 | 15 |
| **Subtotal** | | *2* | | *11* | *18* | *17*[b] |
| **Right** | Likud (LIK) | 40 | 32 | 30 | 19 | 38[b] |
| | Gesher | -- | -- | 2 | -- | -- |
| | Tsomet (TZ) | 2 | 8 | -- | -- | -- |
| | Yisrael beiteinu | -- | -- | -- | 4 | 7 |
| **Subtotal** | | *42* | *40* | *32* | *23* | *45* |
| **Religious** | Shas (SHAS) | 6 | 6 | 10 | 17 | 11 |
| | Yahadut (AI, DH) | 7 | 4 | 4 | 5 | 5 |
| | (Mafdal) NRP | 5 | 6 | 9 | 5 | 6 |
| | Moledet (MO) | 2 | 3 | 2 | 4 | -- |
| | Techiya (TY) | 3 | -- | -- | -- | -- |
| *Subtotal* | | *23* | *19* | *25* | *31* | *22* |
| TOTAL | | *120* | *120* | *120* | *120* | *120* |

[a] Am Ehad or ADL ,under Peretz, combined with Labor, to give the party 19+2=21 seats. .

[b] Olim joined Likud to form one party giving Likud 38+2= 40 seats, and the right 40+7=47 seats

**Figure 1 : Party positions and electoral distribution ( at the 95%, 75%, 50%, and 10% levels) in the Netherlands, based on 1979 data.**

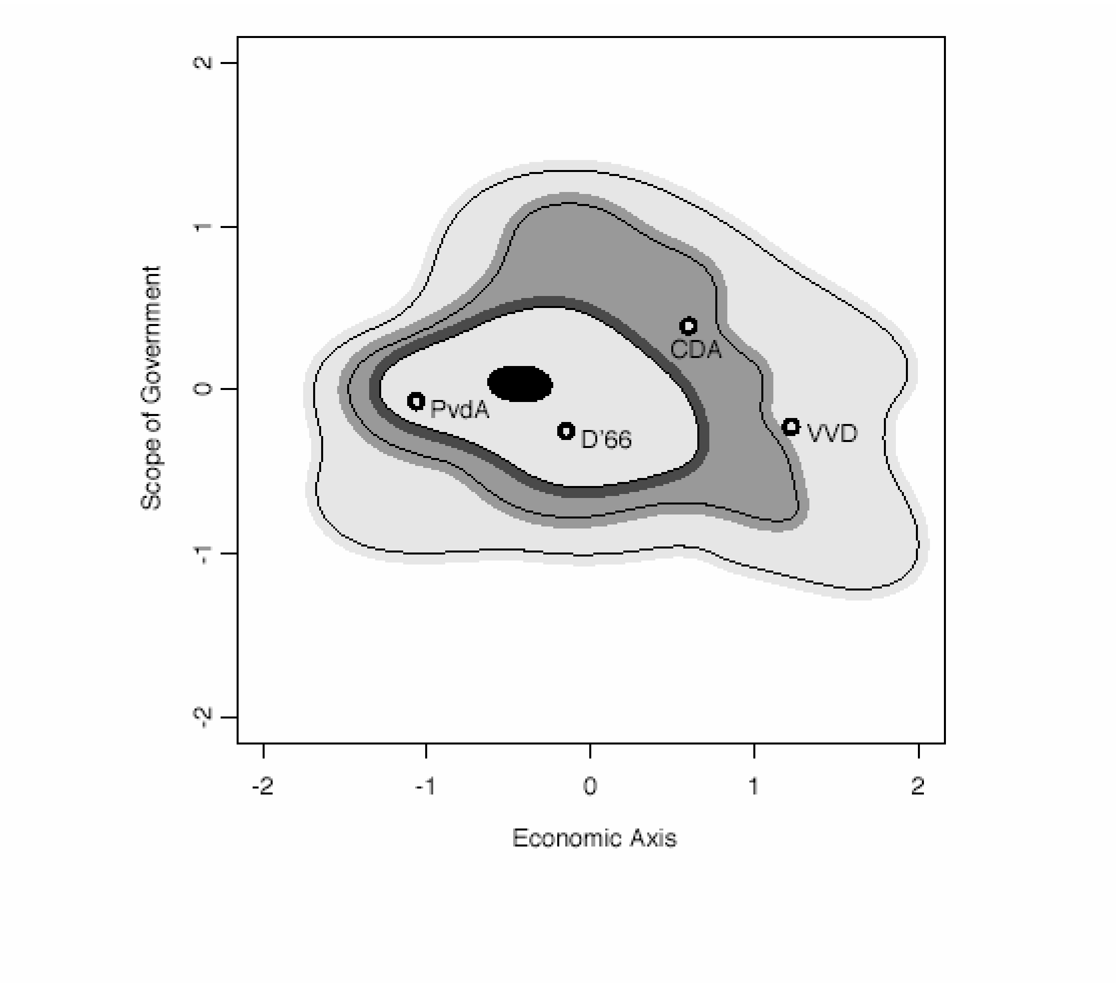


**Figure 2. Coalition Risk  in the Netherlands at the 1981 Election**

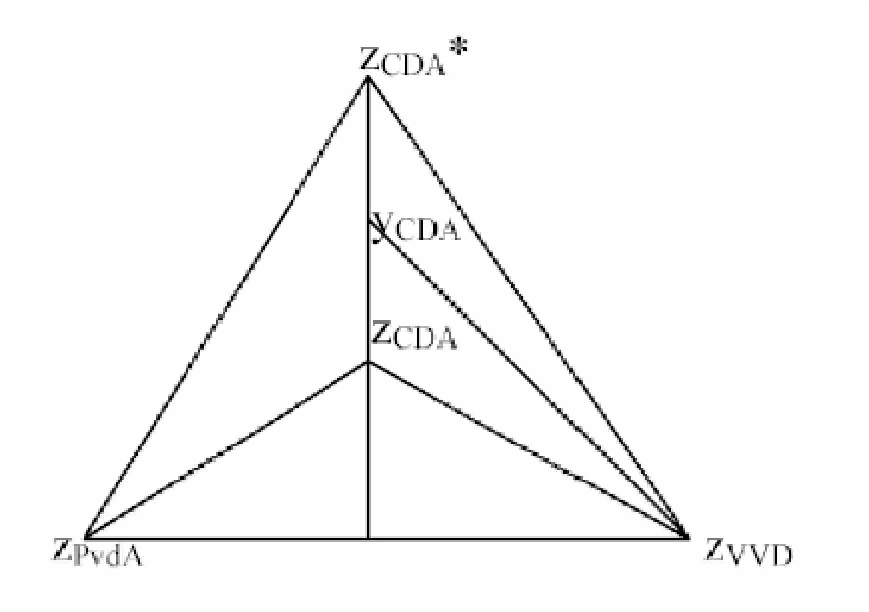


**Figure 3 : Party positions and electoral distribution ( at the 95%, 75%, 50%, and 10% levels) in the Knesset at the election of 1996.**

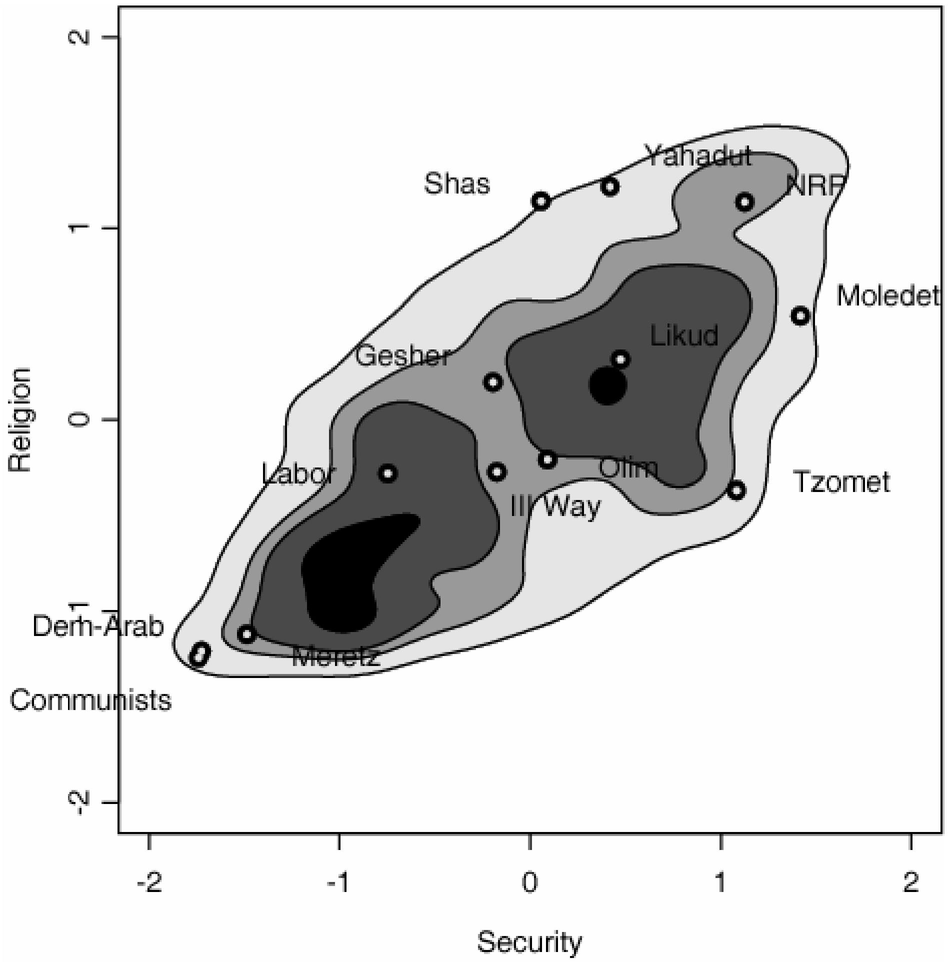
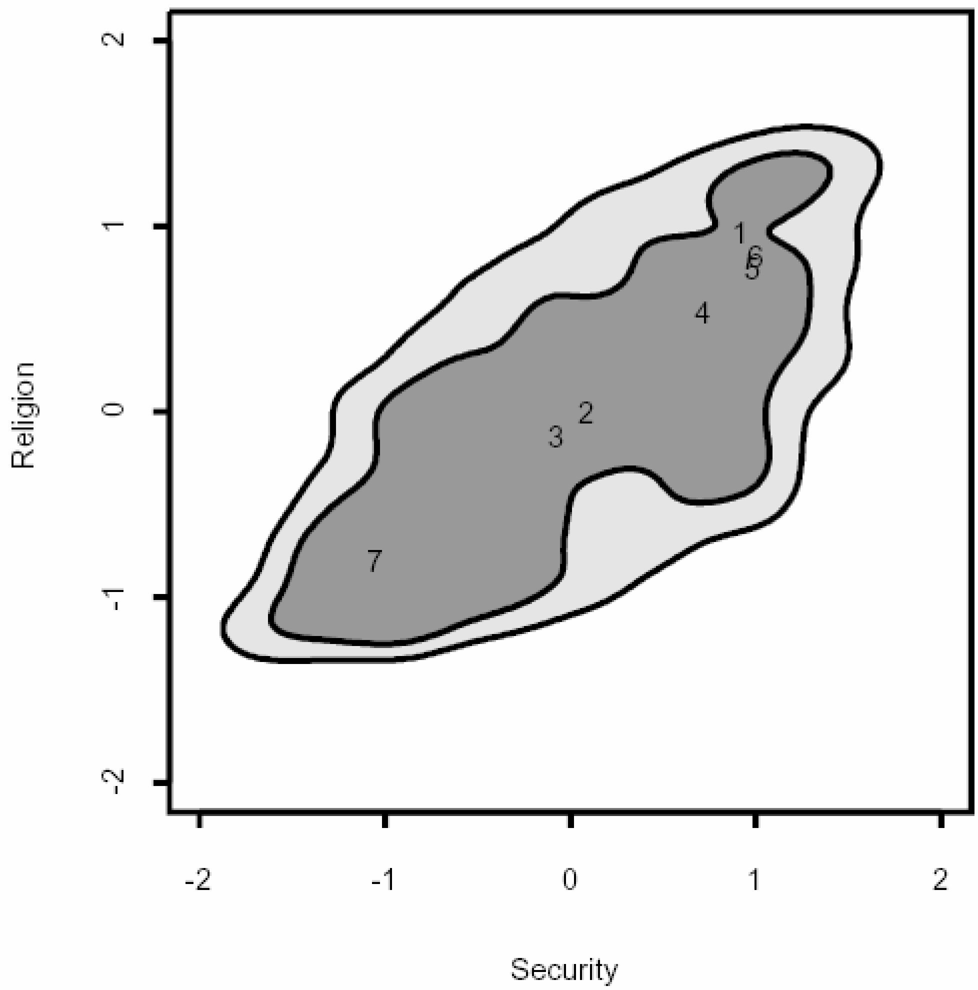


**Figure 4: A representative local Nash equilibrium of the vote- maximizing game in the Knesset for the 1996 election**




Key: 1 = Shas, 2= Likud, 3= labor, 4=NRP, 5= Moledet, 6= III Way, 7= meretz.

**Figure 5 : Party positions and electoral distribution ( at the 95%, 75%, 50%, and 10% levels) in the Knesset at the election of 1992.**

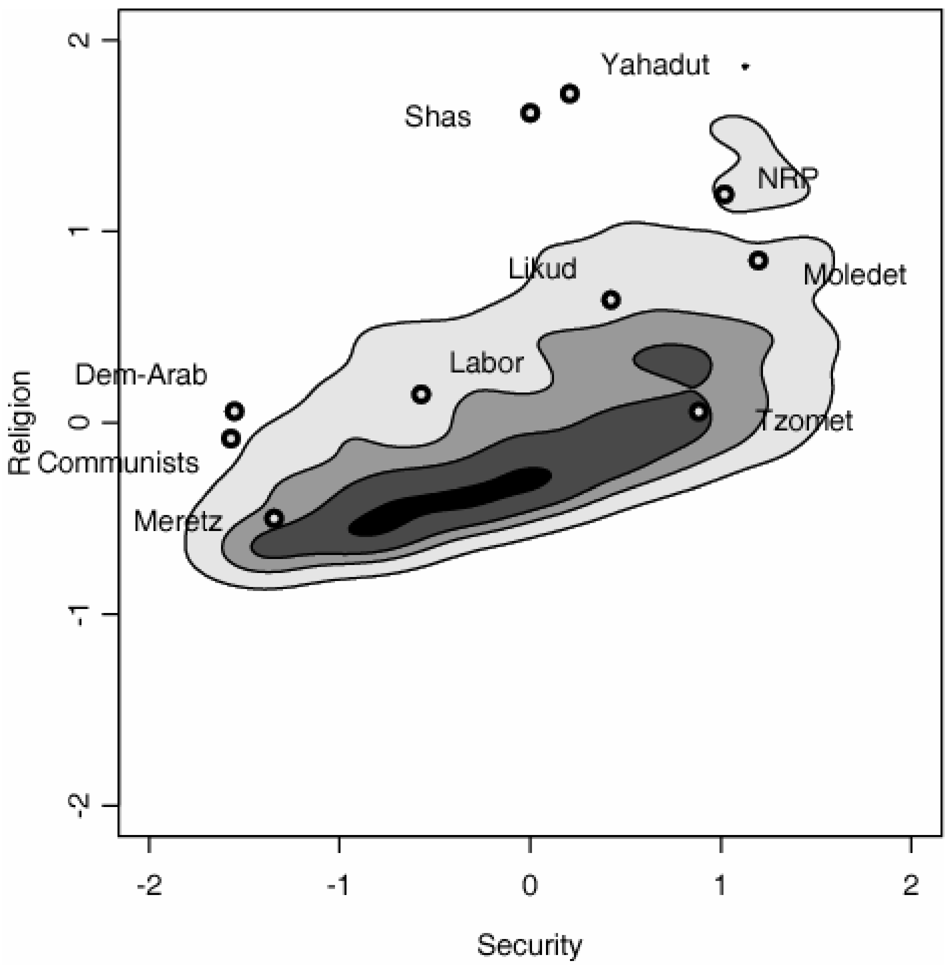
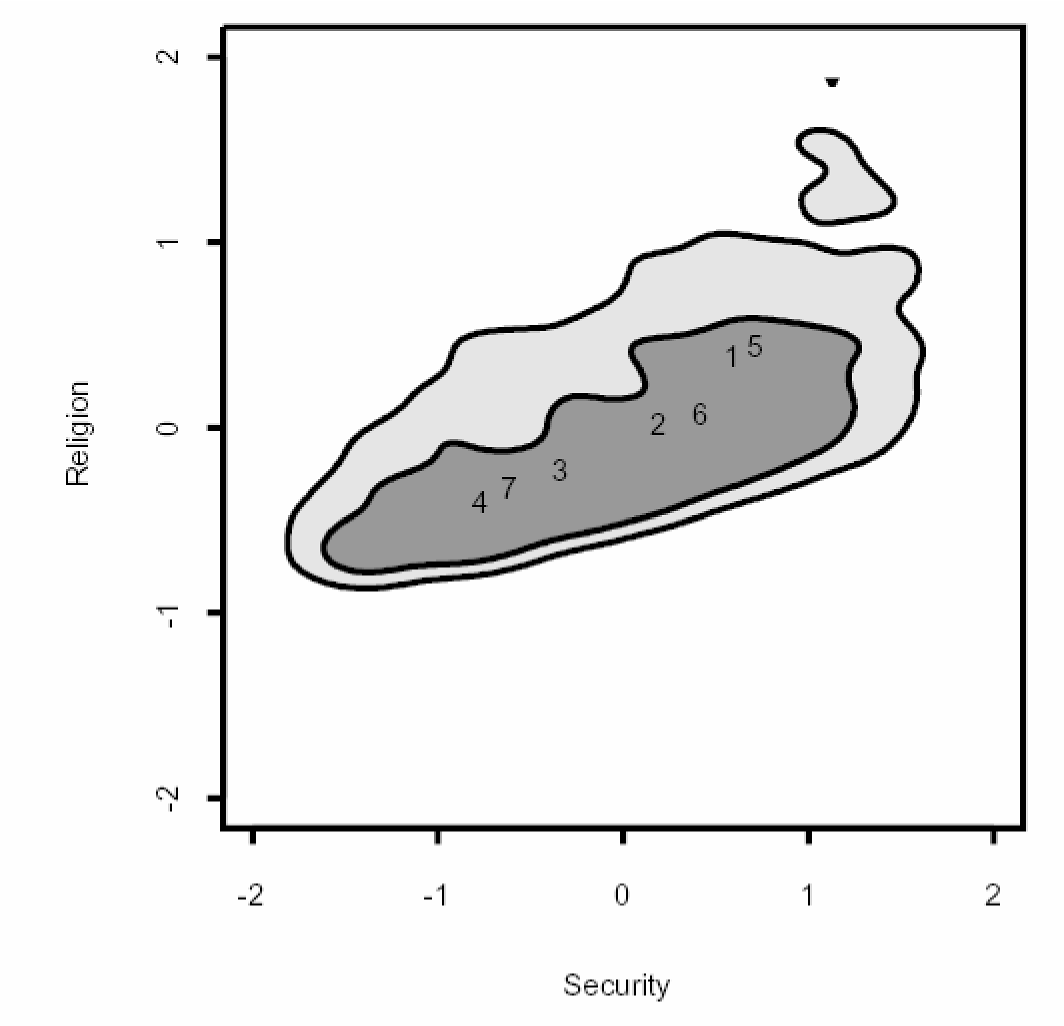


**Figure 6: A representative local Nash equilibrium of the vote maximizing game in the Knesset for the 1992 election.**




Key: 1=Shas, 2=Likud, 3=Labor, 4=Meretz, 5=NRP, 6=Molodet, 7=Tzomet

Figure 7. A schematic representation of the current configuration of the Knesset

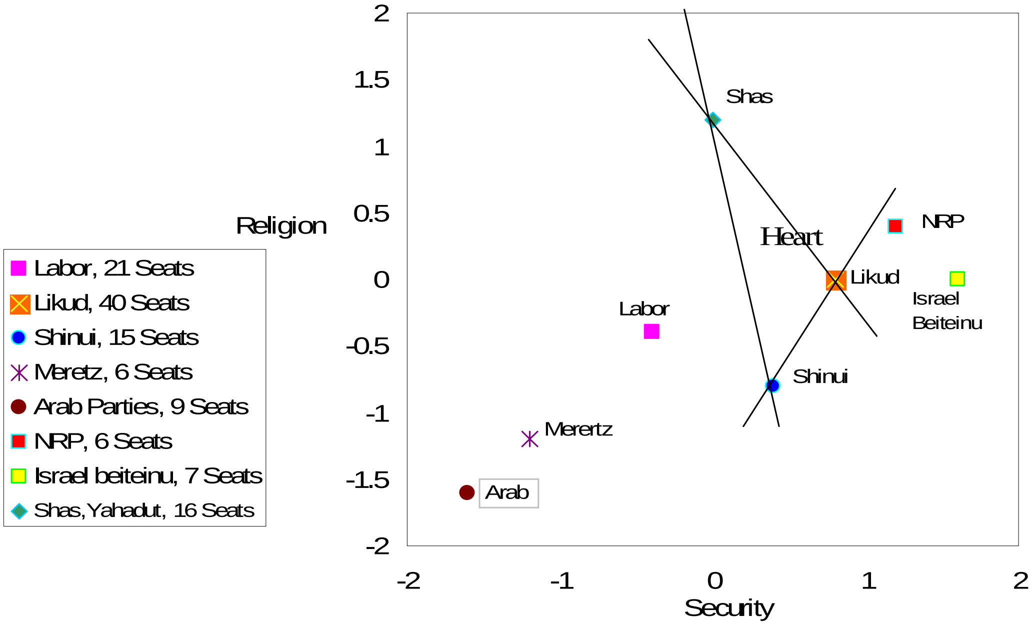


Figure 8. The configuration of the Knesset after  Peretz becomes leader of Labor

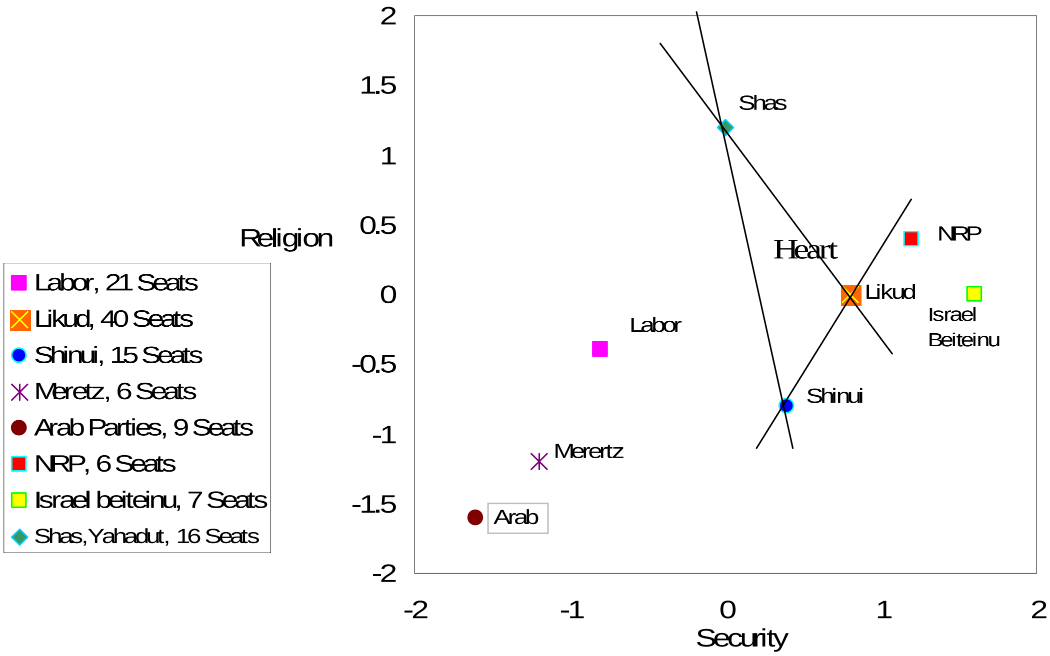


Figure 9: The effect of the creation of Kadima by Ariel Sharon on the configuration of the Knesset

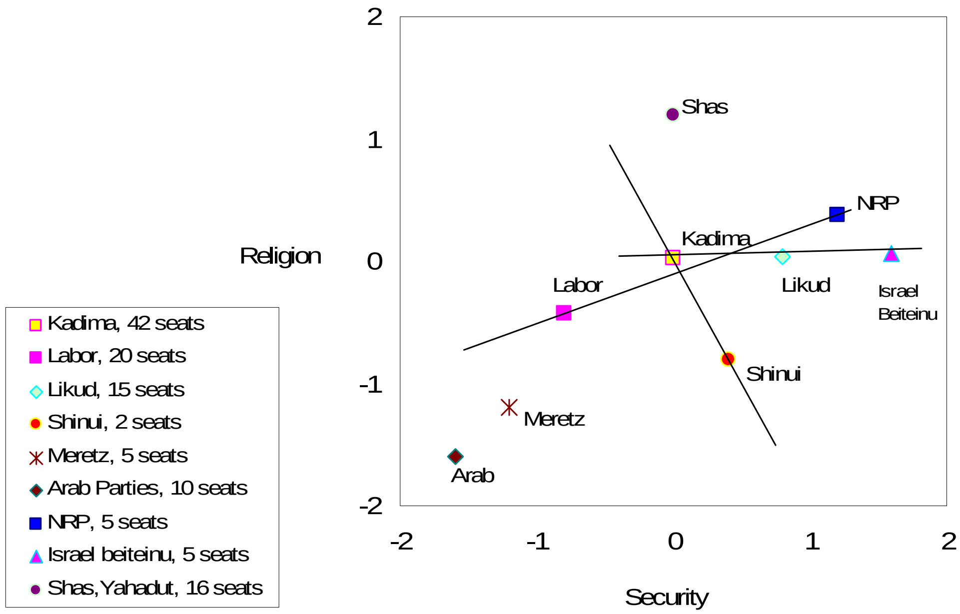
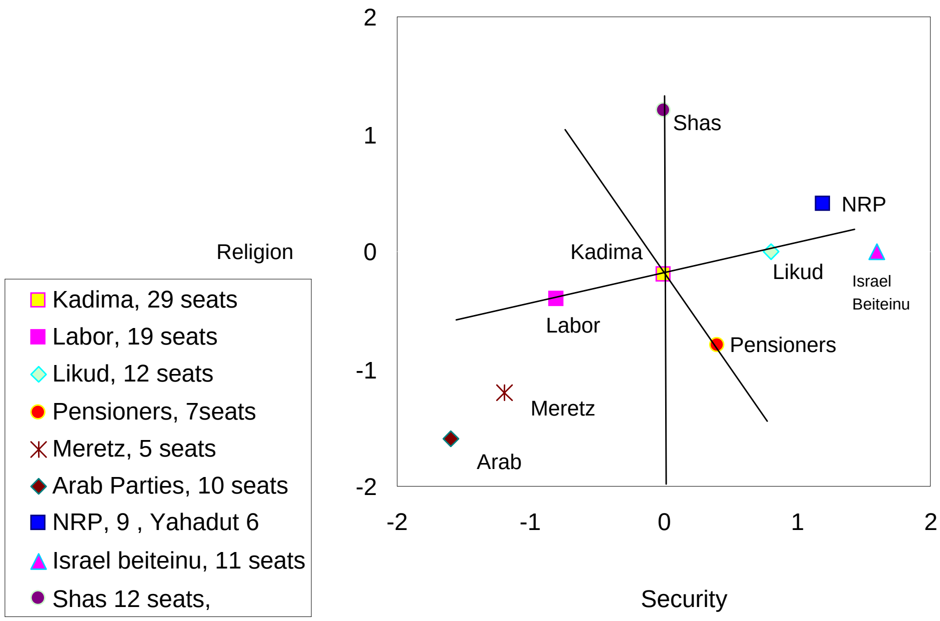


Figure 10. The expected configuration of the Knesset after the March 2006 Election[1]




[1] Seat allocation is based on surveys published in early February, 2006