Heisenberg-limited measurements are of considerable interest for ultrasensitive interferometry.1 Based on bosonic or fermionic quantum interferometry, such measurements possess an ultimate sensitivity limit of $1/n$ (the Heisenberg limit), for a detected photon number $n$, as opposed to the beam splitter’s shot-noise limit (SNL), $1/\sqrt{n}$. In quantum optics the SNL applies whenever the light impinging on the two sides of the input beam splitter of an interferometer comprises one vacuum (unused) mode.2 However, if this vacuum mode is replaced with nonclassical light, such as squeezed vacuum3 or a photon-number correlated (twin) beam,4 the SNL can be exceeded and, for ideal (infinite) squeezing, the measurement becomes Heisenberg limited. Sub-SNL measurements have been demonstrated using vacuum squeezing,5,6 twin-photon pairs,7 and twin pulsed beams,8 all of which involve frequency-degenerate two-beam input into an interferometer.

Another interesting Heisenberg-limited measurement, proposed by Snyder et al.,9 is to use frequency nondegenerate twin beams9,10 for ultrasensitive polarization rotation measurements. The principle of the method is the following: A type II optical parametric oscillator (OPO) above threshold emits intense, orthogonally polarized, photon-number correlated (twin) beams at respective frequencies in narrow bands around $\nu_{1,2}$. A photodetector of bandwidth greater than (or centered at) $|\nu_1 - \nu_2|$ is placed at the OPO output. At first, no ac signal is observed, as the two orthogonally polarized OPO waves cannot interfere. However, if a polarization-rotating medium is inserted, followed by a polarizer aligned with the OPO crystal axes (Fig. 1), then the photodetection signal displays a heterodyne oscillation at $\nu_\perp$ (or beat note) because of the interference of the OPO waves. This signal increases with the polarization-rotation angle. Such a heterodyne measurement can be made at high frequencies and can be as narrowband as the beat-note linewidth. Its signal-to-noise ratio is high, since classical (e.g., $1/f$) noise rolls off at the detection frequency and a narrow detection bandwidth yields a reduced noise floor. It is well known that spectrally improvements can be obtained in ultrasupercise measurements by implementing classical heterodyne techniques.11,12 Thus heterodyne measurements are often shot-noise limited, and quantum optical techniques, such as squeezing, can then be applied to reduce the noise floor further. Going back to our case, Snyder et al.9 predicted that, because of the photon-number correlation of the twin beams within the OPO cold-cavity linewidth $\gamma$, the polarization rotation beat note should rest on a sub-shot-noise floor if $\nu_\perp < \gamma$. For this to occur rotation angle $\theta$ must be small enough that the resulting beam mixing does not destroy the quantum intensity correlation.13 Such a method is therefore well suited to the ultrasensitive detection of minute birefringence. This Letter presents an experimental realization of sub-SNL heterodyne polarimetry.

We first show that this measurement is, ideally, Heisenberg limited. One could indeed wonder whether the Heisenberg limit itself would not be breached by Snyder’s scheme since the ideal twin-beam noise floor is zero, not $1/n$. (The photon-number difference is a constant of motion whose initial state is a twin-vacuum state.) However, a polarization rotation $\theta \neq 0$, in fact, yields no heterodyne signal at all if the twin-beam correlation is ideal. To see this, consider the two outputs of the aforementioned polarizer, $c = a_1 \cos \theta + b_2 \sin \theta$ and $d = a_1 \sin \theta - b_2 \cos \theta$,
where $a_1, b_2$ are the twin input modes at frequencies $\nu_{1,2}$. The number difference is $c^\dagger c - d^\dagger d = \cos 2\theta(a_1^\dagger a_1 - b_2^\dagger b_2) + \sin 2\theta(a_1^\dagger b_2 + a_2^\dagger b_1)$, the second term being the beat note. If one considers a general ideal twin-beam state, $|\psi\rangle = \sum_n c_n |n\rangle_1 |n\rangle_2$ (i.e., a zero noise floor), then the average signal $\langle \psi | c^\dagger c - d^\dagger d |\psi\rangle = 0$, $\forall \theta$, since $\langle n'n'|a_1^\dagger b_2|nn\rangle \propto \delta_{n,n+1}\delta_{n,n-1}$. It is therefore necessary, for a given photon number $n$, to have deviations from perfect photon-number correlation by at least $\pm 1$ photon to observe a nonzero beat-note signal of $\sim n \sin 2\theta$. Hence the smallest detectable angle is $\theta_{\text{min}} \sim \pm 1/(2n)$, of the order of the Heisenberg limit.

In practice, two technical problems must be solved. The first is the residual classical frequency jitter of the beat note, which can mask the noise reduction if the bandwidth of the latter is comparable to the jitter excursion range. Recently, our group achieved above-threshold nondegenerate OPO operation at ultrastable frequency differences (and orthogonal polarizations) in a routinely repeatable way by electronic servo stabilization of the phase difference of the twin beams.\textsuperscript{14} This led to the observation of generalized Hong–Ou–Mandel interference between interfering states with photon numbers much larger than 1 (Ref. 13) and paves the way toward a realization of continuous-variable entanglement of ultrastable bright beams.\textsuperscript{15} When the beat note is provided by our phase-locked OPO (whose frequency difference is ultrastable yet continuously tunable), its jitter is essentially zero.\textsuperscript{13} The second problem is the existence of a large residual beat note, even when $\theta = 0$, because of polarization cross talk in the OPO cavity. If it is not carefully canceled, this residual beat note can reach a few percent of the maximum amplitude, and its pedestal can then easily overwhelm the noise floor. It is thus crucial to suppress this residual beat note. It is known that polarization cross talk in a type II OPO is linked to critical phase matching and walk-off in the OPO crystal.\textsuperscript{16} To suppress polarization cross talk, we noncritically phase match frequency-degenerate parametric oscillation at 1064 nm, pumped at 532 nm, by using a Na:KTPOPO\textsubscript{4} (Na:KTP) crystal.\textsuperscript{17} By carefully aligning the OPO so that the cavity axis coincides with the $x$ principal axis of the crystal, we can obtain a reduction of the residual beat note of more than 65 dB. However, other experimental factors such as wave-plate and polarizer imperfections will also yield a beat note that is just the illustration of the classical sensitivity of heterodyne polarimetry using the perfectly mode-matched OPO beams. Therefore we will not worry about the residual beat note now, as it is small enough for the quantum noise floor to be well resolved.

The experimental setup is depicted in Fig. 1. The OPO consists of a Na:KTP nonlinear crystal, stabilized by a temperature controller at the 0.1-nK level, in which pump photons at 532 nm are downconverted to cross-polarized pairs at 1064 nm. The OPO cavity is formed by mirrors $M_1$ and $M_2$. The 1064-nm twin beams exit through $M_2$ toward the right-hand side of the figure. The reflected depleted pump beam provides the error signal for the cavity lock loop (CLL), and a weak twin-beam leak through $M_1$ is used for the phase-difference lock loop (PDLL). With only the temperature controller and CLL on, the frequency-difference error is $\pm 150$ kHz, because of large Na dopant inhomogeneities in the crystal (the Na proportion is 23%).\textsuperscript{17} Adding the PDLL reduces this error by more than 5 orders of magnitude to less than 1 Hz,\textsuperscript{13,14} while keeping the frequency difference continuously temperature tunable over tens of megahertz. Finally, since the PDLL error signal is obtained from beams leaking through a mirror with very low transmission (Fig. 1), it is entirely classical and the PDLL cannot modify the quantum phase fluctuations.

The OPO is operated at pump-beam powers a few percent above the 50-mW threshold. The twin beams, each with power of 1.4 mW, exit the cavity through $M_1$ and traverse a half-wave plate that can be rotated by $\theta/2$, followed by a polarizing beam splitter aligned with the OPO polarizations. The polarizer is initially aligned without the wave plate so that the residual beat note at $\nu_-$ is minimized. Then the wave plate is inserted and an increase in the beat note is observed, even if $\theta = p(\pi/2)$, because of imperfections. The beat note is temperature tuned to 1 MHz, well within the twin-beam squeezing bandwidth of 5 MHz, and phase locked. The rotation angle is then increased. Figure 2 displays the photocurrent difference power spectrum for $\theta \ll 0.1^\circ$, $\theta = 0.1^\circ$, and $\theta = 1^\circ$. (The beat-note contrast is almost unity because of the near-ideal mode matching of the twin beams coming out of the high-finesse OPO resonator.) The

![Fig. 2. Sub-SNL heterodyne polarimetry signals. In (a)–(c) the two flat traces are the SNL (top) and the detection electronics noise (bottom). The peaked trace is the twin-beam beat-note signal, the subhertz beat-note linewidth,\textsuperscript{13,14} of which is not directly visible on these records, since the spectrum analyzer’s resolution and video bandwidths are 3 kHz (100 averages). The maximum beat-note amplitude ($\theta = 22.5^\circ$) is 25 dBm. MNR, measurement-noise reduction.](image-url)
sub-shot-noise measurement background is clearly visible for small polarization rotations in Figs. 2(a) and 2(b), illustrating the polarimetry sensitivity increase. The maximum noise-floor reduction is $-4.8 \, \text{dB}$ in Fig. 2(a) and in the inset (taking the electronic noise floor into account but not the 94% quantum efficiency of the detectors). Note that the squeezing occurs at frequencies as low as 100 kHz.

The beat note has a noise pedestal that, we surmise, is related to the Schawlow–Townes phase-difference noise of the OPO, as was previously observed for frequency-degenerate twin beams (0-Hz beat note). As $\theta$ increases from 0° to 1°, this noise pedestal grows with the beat note and eventually overwhelms the squeezed background. This noise increase could be caused by contamination by the OPO antisqueezed phase-difference noise as well as vacuum fluctuations, caused by contamination by the OPO antisqueezed background. This noise increase could be with the beat note and eventually overwhelms the frequency-degenerate twin beams (0-Hz beat note).

Several factors ought to be pointed out that will significantly improve the performance. First we are probably limited in the polarization cross-talk cancellation by the strong inhomogeneity of the Na:KTP crystal, which makes it impossible to have a clean separation of the signal and idler propagating in the crystal. Second, residual absorption in the crystal is not as low as what can be achieved in hydrothermally grown or gray-track resistant KTP. Our maximum twin-beam correlation level ($-7 \, \text{dB}$) is position dependent in the crystal and does not match recent records in excess of $-9 \, \text{dB}$.

One ought to bear in mind, however, that noncritical phase matching in the OPO is indispensable here to suppress the residual polarization cross-talk and, therefore, regular KTP is not usable. A possible course of action would be to use periodically poled low-loss ferroelectrics such as hydrothermally grown or gray-track resistant periodically poled KTP.

Finally, we compare this measurement with previous realizations of sub-SNL interferometry, using squeezed vacuum, twin-photon, and twin-pulse inputs. We believe that our measurement has the largest noise reduction to date. Moreover, in all other measurements, the two input fields are indistinguishable (frequency degenerate), whereas here the fields are frequency nondegenerate and clearly distinguishable. The heterodyne nature of the signal thus makes it easy to escape classical and $1/f$ noise. Because of the exquisite degree of control that our ultrastable cw OPO displays, we believe that further improvement of our measurements beyond $-4.8 \, \text{dB}$ is well within reach.

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